OPERATING CHARACTERISTICS OF GROUND-WATER RESERVOIRS OCCUPYING A TRENCH

By

Robert E. Glover
Engineering Consultant

and

Morris M. Skinner
Assistant Research Engineer

Department of Civil Engineering
Colorado State University
Fort Collins, Colorado

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ABSTRACT

Sediment filled valleys of intermittent streams can be used as underground reservoirs for storage of water. Water so stored is not subject to evaporation losses and the seepage losses may be small. Recharge and operational characteristics of such reservoirs are expressed in formula form. Some comments are made in regard to the problems of water quality maintenance.

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In the western part of the United States there are many sediment filled stream valleys. In a previous geologic era rainfall must have been more plentiful than it is now and the streams excavated their valleys to depths much below present surface elevations. As the climate became more arid the stream flows were reduced or became intermittent, the ability to transport sediment was likewise reduced and the valleys were filled to present levels with sediment that these streams were able to move in times of flood. Present day streams or watercourses often follow closely the valleys they occupied in the previous geologic era. In the South Platte River Basin in Colorado, examples of sediment filled trenches are provided by the present valleys of Lost, Kiowa, Bijou, and Beaver Creeks [1]. These particular valleys have sediment fills that range in width from 2 to 7 miles, and in depth up to nearly 200 ft. Some idea of the capacities of the ground-water reservoirs provided by these sediment-filled valleys can be obtained from the following table:

Table 1. Volumes of certain ground-water reservoirs occupying sediment-filled valleys in the South Platte River Basin [1]

Valley	Quantity of ground water in storage (acre-feet)
Lost Creek (Prospect Valley)	940,000
Kiowa and Bijou Creeks	2,884,000
Beaver Creek	584,000

These volumes may be compared with the total volume of 13 major surface reservoirs in the South Platte River Basin. The total storage capacity of these surface reservoirs is about 548,000 acre-feet [2].

Surface reservoirs are subject to heavy losses by evaporation and as the available water supplies become more and more heavily encumbered it becomes of interest to investigate the possibilities of improving the water supply by storing water underground where these heavy losses do not occur. If it also happens that the water so stored can be readily recovered and that seepage losses are low, then the possibilities of the use of ground-water reservoirs for improving the water supply become attractive. An evaluation of these possibilities has been made in the case of the Prospect Valley area in Colorado where ground water is a vitally important source of supply and the recharge and recovery of ground water for irrigation has become an established practice. In this case, the ground water is stored in the sediments filling a trench below the present valley of Lost Creek. Some ground water recharge to the Lost Creek aquifer occurs naturally from deep percolation losses incidental to irrigation and from precipitation. Additional recharge has resulted from intermittent deliveries to Olds Reservoir during the past years.

Fig. 1. An aerial view of Olds Reservoir looking down-valley to the northeast towards the South Platte River.

Olds Reservoir, with a capacity of about 400 acre-feet, was originally intended for surface storage, but was soon abandoned due to high leakage losses. This Reservoir, shown in fig. 1, is ideally situated for recharge purposes and with the development of pumping for irrigation in Prospect Valley it has been used for a ground-water recharge site. Water is supplied to Olds Reservoir through a canal system diverting flow from the South Platte River below Denver, Colorado. Since 1938, records show that approximately 40,000 acre-feet have been supplied to the Lost Creek aquifer through Olds Reservoir. Experience has shown that Olds Reservoir will recharge the underlying ground-water reservoir at the rate of at least 35 cubic feet per second. This is equivalent to approximately 70 acre-feet per day and represents about one-foot of depth per day over the surface area of the reservoir. In one period of about eleven months during 1957 - 1958 approximately 12,000 acre-feet were delivered to the Lost Creek ground-water reservoir by use of this facility. The response of the ground-water reservoir to the various recharge and withdrawal amounts over the years has been demonstrated by water level measurements in observation wells.

When nature has provided an under-ground reservoir of this type and suitable spreading areas for recharge are available, or can be arranged, it becomes pertinent to inquire into the operating characteristics of such a reservoir. Among the questions to be answered are the following:

(1) How can the water be recovered?

(2) With reference to a proposed recharge area, when and in what amounts will the recharged water become available to users at specified locations?

(3) What leakage losses will be incurred?

(4) How much of the recharged water can be recovered by the owners of the recharged water?

(5) How will the use of an under-ground reservoir affect the quality of the stored water?

Recovery of water from an under-ground reservoir is generally accomplished by pumping. Operation of the pump will cause a lowering of the water table around the well. The magnitude of the lowering can be estimated by using the formula

$$y = \frac{Q}{2\pi KD} \int_{\frac{r}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du \qquad (1)$$

where

y	represents the drawdown at the radius r at the time t .
Q	the flow of the well.
K	the permeability of the aquifer from which the pump draws water.
D	the original saturated thickness of the water-bearing stratum and
α =	$\frac{KD}{V}$

Where V represents the ratio of the drainable or fillable voids to the total volume. The quantity u is a variable of integration. The integral in the above expression has been tabulated [9] [10]. Formula (1),as well as other formulas presented later, has been prepared for use with consistent units. A consistent system in this sense permits only one unit of a kind. In English units the foot and second are suitable.

Recharge operations create a mound in the water table under the recharge area. From the standpoint of the ground-water user, recharge water has reached him if it has brought or maintained the water table to a suitable level at his well. To answer the question as to when the recharge water will reach a user it is necessary to follow the spreading of the ground-water mound. If the recharge area can be idealized as circular with a uniform rate of recharge over a circular area of radius a then the spreading of an instantaneously generated circular mound of height H under the recharge area is given by the integral [6]

$$h = H \frac{e^{-\frac{r^2}{2\sigma^2}}}{\sigma^2} \int_0^a e^{-\frac{\rho^2}{2\sigma^2}} I_o \left(\frac{r\rho}{\sigma^2}\right) \rho d\rho \qquad (2)$$

where

h represents the height of the mound at the radius r at the time t

H the height of the mound over the circle of radius at a time zero

$\sigma = \sqrt{2 \alpha t}$

a the radius of the recharge area

ρ represents a radius running between the limits 0 and a

$I_o \left(\frac{r\rho}{\sigma^2}\right)$ represents the modified Bessel function of zero order and argument $\left(\frac{r\rho}{\sigma^2}\right)$

It may well be asked why, since the recharge will ordinarily be continuous, a solution is presented in terms of an instantaneously generated mound of height H . The answer to this question is to be found in mathematical difficulties. These difficulties will be appreciated if it is explained that an integration of expression (2) with respect to time would produce the solution appropriate for a continuous recharge. The solution (2) will serve very well, however, if the continuous recharge is replaced by a series of equal increments originating at evenly spaced time intervals and the effects of each increment are added. The integral

$$P\left(\frac{a}{\sigma}, \frac{r}{\sigma}\right) = \frac{e^{-\frac{r^2}{2\sigma^2}}}{\sigma^2} \int_0^a \rho e^{-\frac{\rho^2}{2\sigma^2}} I_o\left(\frac{r\rho}{\sigma^2}\right) d\rho \qquad (3)$$

occurs in a number of physical situations and has been evaluated and tabulated. It is known as the P function [4] [6]. The quantity P is expressed as a function of the parameter $\frac{a}{\sigma}$ and $\frac{r}{\sigma}$. Then, in this terminology:

$$\frac{h}{H} = P\left(\frac{a}{\sigma}, \frac{r}{\sigma}\right) \qquad (4)$$

Beyond a radius of 2.5a equation (1) can be used to represent the rise of ground water levels due to a continuous recharge of amount Q.

When recharge and pumping go on simultaneously their effects at any given point can be evaluated, to a first approximation, by computing the two effects separately and adding the results together. This superposition process will yield good approximations so long as the quantity h/D remains small compared to unity.

A valley gradient γ will generally be present. This will cause a down-valley movement of a mound or a cone of depression at the rate $\gamma K/V$. In the Prospect Valley area this comes out to be about a quarter of a mile per year. This factor can be accounted for by making an allowance for this down-valley movement.

Expressions (1) and (2) imply that the aquifer is of infinite extent. In the case of flow in a trench this will not be the case and it will be necessary to account for the presence of impermeable boundaries. This can be done by the use of images. As an illustration of this method it may be assumed that there is an impermeable boundary near the pumped well. This boundary will impose a condition of no flow. In the assumed infinitely extended aquifer this condition can be imposed by marking out the boundary on the infinite aquifer and introducing an image well directly opposite the original pumped well. This will impose the condition of no flow along the entire boundary if both are pumped wells. The image well, of course, has no real existence. Its use is a mathematical ruse to meet a boundary condition.

During the latter part of 1959 and first part of 1960 a field study was undertaken in Prospect Valley to compare the above theory with actual field measurements. Approximately 9400 acre-feet were recharged through Olds Reservoir at a fairly constant rate of about 70 acre-feet per day. The dissipation of the recharge-water mound was observed by frequent water-level measurements in the surrounding observation-well network. The observed arrival of the recharge water at the various observation wells was in close agreement with the theoretical dissipation of the recharge mound beneath Olds Reservoir [6].

Another type of solution is useful for estimating leakage from the ground-water reservoir. This is the line source solution

$$y = \frac{q_1 x}{2\pi KD} \sqrt{\pi} \int_{\frac{x}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u^2} \, du \qquad (5)$$

where y represents the drawdown produced at the distance x from the origin at the time t due to the removal of the flow q, per unit length along the line $x = 0$. In English units the dimensions of q is cubic feet per second per foot of line, or square feet per second. It is assumed that the flow q comes from both sides of the origin. This solution becomes indeterminate at the origin and must there be replaced by

$$y_o = \frac{q_1 \sqrt{4\pi\alpha t}}{2\pi KD} \qquad (6)$$

The integral appearing in formula (5) has been tabulated by M. W. Bittinger [8]. A copy of this table is included.

The flow across a plane at the distance x from the origin is given by the expression

$$f_1 = \frac{q_1}{2} \left[1 - \frac{2}{\sqrt{\pi}} \int_0^{\frac{x}{\sqrt{4\alpha t}}} e^{-u^2} du \right] \tag{7}$$

This expression can be used where the point of escape is at a distance from the recharge and use areas which are large compared to the width of the trench. The recharge and pumping can then be idealized as line sources and sinks respectively and the flows at the point of escape can be estimated. Images will be useful here, also, to meet the appropriate boundary conditions [10].

In the Prospect Valley area, for example, the point of escape is at the junction of the valley of Lost Creek and the South Platte River. The recharge area is about 15 miles distant from the South Platte and the area of pumping is at an average distance of 12 miles from the River. Both of these points are south of the river. Flows to or from the south are blocked by the configuration of the impermeable beds at a point about one mile south of the Olds Reservoir recharge area. Images are used to account for the presence of this barrier. The source and sink and their images are imaged in a line at the junction of Lost Creek and the South Platte River where water levels are maintained. The seepage losses estimated in this way are such that water recharged from Olds Reservoir and stored in year <u>one</u> and pumped in year <u>ten</u> would sustain only about a 9 per cent loss.

The 9 per cent loss estimate does not include the normal ground-water under-flow due to the down-valley gradient of the water table. In the Prospect Valley area the flow through the aquifer due to the valley gradient is estimated to be about 11 cubic feet per second [1]. This flow is beneficial since if it were not present there would be no way to get rid of the accumulation of salt originating from the use of irrigation water applied to the land. Results of limited ground-water quality determinations for Prospect Valley, as shown in Table 2, illustrate an increase in total solids in the ground-water with distance down valley.

Table 2. Total solids concentration in the ground water in the Lost Creek aquifer

Distance Downvalley from Olds Reservoir (miles)	Total solids
1.0	441 (6-10-60)
2.0	646 (4-27-60)
3.0	1006 [7]
7.0	2191[7]

Where the irrigated area overlies the reservoir, as is the case in Prospect Valley, irrigation water can be used with unusual efficiency. In areas supplied by surface diversion the irrigator is generally unable to recover the water lost by deep percolation. When the ground-water reservoir underlies the irrigated area, however, the deep percolation losses return to the reservoir where it is available for reuse. Where soils are light and deep percolation losses relatively high, this can be an important factor.

The following development relates the total irrigation use to the original recharge. Suppose a recharge amount R is stored in the ground-water reservoir and is subsequently pumped to the surface and used for irrigation where the fractional part n is lost by deep percolation. The part lost can again be pumped and applied. The sum S of all of these pumpages will be

$$S = R\,(1 + n + n^2 + n^3 \ldots\ldots\ldots\ldots\ldots) \qquad (8)$$

If both sides of this equation are multiplied by n the result is:

$$n\,S = R\,(n + n^2 + n^3 + n^4 \ldots\ldots\ldots\ldots\ldots)$$

A subtraction yields the relation

$$S - nS = R$$

or

$$S = \frac{R}{1 - n} \tag{9}$$

This expression indicates the total amount pumped when all of the original recharge has finally been consumed. The ratio of the total pumpage S to the original recharge amount R is indicated in the following table:

Table 3. Ratios of pumpage to recharge.

Part lost n	$\frac{S}{R}$	$\frac{S_1}{R}$
0.1	1.111	1.111
0.2	1.250	1.248
0.3	1.428	1.417
0.4	1.667	1.624
0.5	2.000	1.875

If the water is applied four times then formula (9) is replaced by:

$$S_1 = R \frac{(1-n^4)}{(1-n)} \tag{10}$$

Values of the ratio $\frac{S_1}{R}$ are also shown in the above table.

In the South Platte area it would be reasonable to expect a diversion of 2.0 acre-feet of water for each acre irrigated through one season and an actual consumptive use of one acre-foot on each acre in this period. If the deep percolation loss is equal to the consumptive use, then $n = 0.5$. In this case, assuming at least four uses, somewhere between 1875 and 2000 acre-feet could be pumped for irrigation purposes for each 1000 acre-feet put into ground-water storage. The ground-water user is therefore in a position to make a much more efficient use of his water supply than is the irrigator supplied from surface diversion.

Since a ground-water reservoir is not readily emptied or flushed out, contamination of a ground-water reservoir, either through incidental or intended recharge, can be a serious matter. Surveillance and necessary control measures on the quality of recharge water should be exercised to prevent the introduction of external contaminants. Constituents, contained in recharge waters, which may react or combine within the aquifer to produce harmful results also need to be eliminated.

Conclusions:

(1) The many sediment-filled stream valleys in the western part of the United States afford possibilities for development of underground storage reservoirs.

(2) With due consideration for boundaries, aquifer characteristics, recharge and withdrawal amounts, the use of theoretical expressions have proven to be quite reliable for defining the operating characteristics of the ground-water reservoir in the sediment-filled trench of Prospect Valley. It is logical to assume that the operating characteristics of sediment-filled trenches in similar areas may be equally as well defined.

(3) Underground reservoirs can provide storage not subject to evaporation or phreatophyte losses, whose volume is not decreased by silt deposits, whose leakage may be small, which does not incur land loss due to flooding, whose construction costs are moderate and from which water can be obtained with a minimum of delivery complications.

(4) An underflow is needed to prevent accumulation of salt and will generally be present because of the valley gradient.

(5) Underground reservoirs require management methods which differ from those used for surface reservoirs. These methods can be based upon the measured amounts of recharge and draft and the properties of the aquifer.

(6) Where the underground reservoir underlies an area irrigated by pumping, unusual opportunities are afforded for the efficient use of water.

(7) A combination of deep surface reservoirs at high altitudes, where evaporation is naturally low, and underground reservoirs at the lower altitudes provides an effective combination for holding evaporation losses to a minimum.

(8) Considerable attention should be given to insuring that recharge supplies to a ground-water reservoir be of suitable quality.

(9) Additional research needs to be accomplished to better understand the physical, chemical and economic factors involved in managing our ground-water reservoirs most efficiently.

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Table $$I_x = \sqrt{\pi}\int_{\frac{x}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u^2}\,du$$

$x/\sqrt{4\alpha t}$	I_x	$x/\sqrt{4\alpha t}$	I_x	$x/\sqrt{4\alpha t}$	I_x
0.0000	∞	0.34	2.6628	0.78	0.38848
0.0005	3541.8	0.35	2.5306	0.79	0.37294
0.001	1769.3	0.36	2.4065	0.80	0.35804
0.002	883.07	0.37	2.2901		
0.003	587.68	0.38	2.1805	0.81	0.34373
0.004	439.98	0.39	2.0774	0.82	0.33000
0.005	351.36	0.40	1.9802	0.83	0.31681
0.006	292.28			0.84	0.30415
0.007	250.08	0.41	1.8885	0.85	0.29199
0.008	218.43	0.42	1.8018	0.86	0.28032
0.009	193.81	0.43	1.7199	0.87	0.26911
0.01	174.12	0.44	1.6424	0.88	0.25834
		0.45	1.5689	0.89	0.24800
0.02	85.516	0.46	1.4993	0.90	0.23807
0.03	55.993	0.47	1.4333		
0.04	41.241	0.48	1.3706	0.91	0.22853
0.05	32.396	0.49	1.3110	0.92	0.21936
0.06	26.506	0.50	1.2544	0.93	0.21056
0.07	22.303			0.94	0.20210
0.08	19.156	0.51	1.2005	0.95	0.19397
0.09	16.712	0.52	1.1493	0.96	0.18616
0.10	14.760	0.53	1.1004	0.97	0.17866
		0.54	1.0539	0.98	0.17146
0.11	13.166	0.55	1.0096	0.99	0.16453
0.12	11.841	0.56	0.96728	1.00	0.15788
0.13	10.722	0.57	0.92692		
0.14	9.7661	0.58	0.88840	1.1	0.10414
0.15	8.9397	0.59	0.85162	1.2	0.06820
0.16	8.2186	0.60	0.81647	1.3	0.04426
0.17	7.5845			1.4	0.02843
0.18	7.0227	0.61	0.78289	1.5	0.01806
0.19	6.5219	0.62	0.75078	1.6	0.01133
0.20	6.0728	0.63	0.72008	1.7	0.00702
		0.64	0.69070	1.8	0.00429
0.21	5.6682	0.65	0.66260	1.9	0.00259
0.22	5.3018	0.66	0.63570	2.0	0.00154
0.23	4.9688	0.67	0.60994		
0.24	4.6650	0.68	0.58527	2.1	0.00090
0.25	4.3868	0.69	0.56164	2.2	0.00052
0.26	4.1313	0.70	0.53900	2.3	0.00029
0.27	3.8959			2.4	0.00016
0.28	3.6785	0.71	0.51730	2.5	0.00009
0.29	3.4772	0.72	0.49651	2.6	0.00005
0.30	3.2905	0.73	0.47657	2.7	0.00003
		0.74	0.45745	2.8	0.00001
0.31	3.1168	0.75	0.43912	2.9	0.00001
0.32	2.9550	0.76	0.42153	3.0	0.00000
0.33	2.8040	0.77	0.40466		

Computed from National Bureau of Standards, Tables of Probability Functions, Vol.I, MT8, U.S. Government Printing Office, 1941.