MOTION OF SINGLE PARTICLES IN SAND CHANNELS

By

Neil S. Grigg

Open-file Report
June 1969

CER68-69NSG-42

UNITED STATES
DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY
Water Resources Division
Fort Collins, Colorado

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ABSTRACT

The motion of single particles over ripple and dune beds was investigated in a laboratory flume. Statistical properties of the step lengths, rest periods and bed profiles were calculated and analyzed. Seven different runs were made to detect the variation of particle motion with hydraulic conditions. Two bed material sizes were used.

The investigation was made possible through the use of single high-activity radioactive particles. Medical tracer particles with specific gravity of approximately 2.65 were used. Median diameters were approximately 0.33 and 0.45 mm. In general, the tracers proved quite satisfactory for studies of this type.

The measurement of the statistical properties of particle motion made possible the evaluation of a general two-dimensional stochastic dispersion model developed by Sayre and Conover (1967). It was found that the model predicted well the mean velocity of the centroid of a group of particles but underpredicted the rate of spreading of their concentration distribution. Other observations suggested that the assumptions made in the development of the model are too restrictive.

The particle step lengths followed the gamma distribution and the rest periods followed the exponential distribution. Parameters of

the distributions related to flow conditions in a predictable manner. The distribution of bed form lengths followed a gamma distribution indicating a relation between step length and bed form length. The relation is most evident in the dune range. Good correlations between particle motion, hydraulic conditions and bed properties give promise that bed-load movement can be predicted either from a knowledge of hydraulic conditions or the statistical properties of the sand waves.

The probability distribution for elevations of deposition is investigated through an equation relating conditional and unconditional rest periods. These investigations, along with other observations made during the tests, suggest that most deposition and erosion occurs below the mean bed elevation. Some inferences are made from this deduction about the detailed processes of sediment transport and bed form migration.

The results of the study show that previously developed simple models may not be adequate to fully describe particle motion. The experimental techniques developed can be used to study further the mechanism of particle motion to define in a more general fashion the relations between sediment motion, sediment properties and hydraulic conditions.

ACKNOWLEDGMENTS

Sincere appreciation is due the American taxpayer who has financed practically all of the writer's undergraduate and graduate education.

The study was part of the U.S. Geological Survey Water Resources Division research program on mechanics of flow and sediment transport in alluvial channels. The writer acknowledges the financial support of the Geological Survey as well as the Department of Civil Engineering, Colorado State University, in the form of a Graduate Research Assistantship.

Several persons deserve credit and gratitude for their participation in the study. In particular the members of the graduate committee, Professors D. B. Simons, E. V. Richardson, H. J. Koloseus, P. W. Mielke, Jr. and R. E. Gaines are extended a vote of thanks. C. F. Nordin, Jr. and W. W. Sayre deserve credit for their encouragement and assistance during the study. Mrs. Lois Niemann is commended for her work in the machine computation necessary to process the data. Mrs. Joyce Gross receives gratitude for her excellent work in the preparation of the manuscript. Jamshid Safai is due appreciation for his assistance in the reduction of the basic data. The guidance of T. Yang in the early stages of the data collection program is acknowledged.

Finally the writer acknowledges with gratitude the influence of his family who by their encouragement are largely responsible for the attainment of his educational goal. In particular Dr. and Mrs. R. I. Vachon receive credit for their constant encouragement in the early stages of his graduate work, his wife and children are due sincere gratitude for their patient sacrifice during the graduate program and his mother receives gratitude for her constant support.

LIST OF SYMBOLS

Symbol	Definition	Dimensions
A, B, C, D	Constants	
$b(y)$	A function of y	
d_i	Grain diameter for which i percent by weight are finer	L
d_s	Characteristic diameter of a particle	L
$E(\cdot)$	Expected value	
$F(\cdot)$	Distribution function	
$f(\cdot)$	Density function or function g	
$f^{(n)}(\cdot)$	n-fold convolution of a probability density function	
g	Acceleration of gravity	L/T^2
$H(t)$	Fixed bed level of $y(t)$ record in standard deviations	
I	An integral	
k_1	Parameter of gamma probability distribution	L^{-1}
k_2	Parameter of gamma probability distribution	T^{-1}
$N(t)$	Counting function	
n	A particular number of steps or rest periods	
q_s	Rate of sediment discharge	F/TL
r	Shape parameter of gamma distribution	
s_x^2	Variance of step length distribution	L^2
T	Random variable denoting the length of a rest period	T
t	Measure of total time	T
X	Random variable denoting the length of a step	L

Symbol	Definition	Dimensions
U_*	Fluid friction velocity	L/T
x	Measure of longitudinal distance	L
Y	Random variable denoting an elevation of deposition	L
y	Measure of vertical distance	L
Z	A random variable	
α, β	Constants	
$\Gamma(\cdot)$	Gamma function	
γ_s	Specific weight of sediment	F/L^3
δ	Thickness of layer of motion	L
ε	A very small number	
$\lambda(\cdot)$	A probability function	L^{-1}
μ	A mean value	
ν	Kinematic viscosity of water	L^2/T
ρ	Density of fluid	FT^2/L^4
ρ_s	Density of sediment	FT^2/L^4
σ	Standard deviation	
τ_*	Shields' dimensionless shearing stress	
τ_c	Critical shearing stress	F/L^2
$\phi(\cdot)$	A function	
ω	Particle fall velocity	L/T

Chapter I

INTRODUCTION

Sediment transport is the result of the motion of many individual particles. Each particle moves in response to forces which vary in a seemingly random fashion. To date, no theory has been able to predict accurately quantities of transport under all conditions. The most reliable methods of prediction seem to be those based on empirical data observed under a narrow range of conditions.

The need to understand the mechanics of sediment transport is quite real. Our watersheds constantly yield enormous quantities of sediment to their streams. Usually the sediment is removed from points where it is needed, such as from highway fill material or new building developments, to undesirable places such as storage reservoirs. Extremely valuable reservoir storage space is continuously being filled with useless sediment. This problem is particularly acute in the Western United States where water storage is dear and sediment plentiful.

In addition to the transport of sediments there is a need to understand the dispersion process. A direct application of such understanding is the calculation of allowable quantities of radioactive waste to be placed into streams. One must know how long it takes for certain quantities to be dispersed below initial concentration levels.

Finally, the movement of sediment is a basic process to the forming and shaping of the earth's features. It is of basic scientific as well as engineering interest to understand the building of beaches, the migration of sand dunes, the formation of river deltas and the movement of sand in a storm.

Anyone who has had an opportunity to observe closely the movement of sediment knows that each grain moves in a series of steps and rests. The steps and rests vary in magnitude according to external conditions. The frequencies and magnitudes of these steps and rests are of basic interest in understanding the nature of the movement of the sediments.

This study is intended to complement earlier transport-dispersion studies by Hubbell and Sayre (1964), Sayre and Conover (1967) and Yang (1968). It was desired to make a Lagrangian study of particle motion in conjunction with the series of dispersion studies. Experimental difficulties prevented such a study until recently.

Heretofore, it has not been possible to measure, except under very limited circumstances, the magnitudes of the step lengths and rest periods. The difficulty was the tracing of a particle to measure its motion. In this present study, a special preparation of radioactive tracers was made with sufficient activity affixed to each particle so that it could be traced. The particle could then be followed to measure accurately the length of each step and the duration of each rest period.

These experimental techniques permit exploration into areas never before analyzed. The knowledge of the basic parameters allows the evaluation of mathematical models for transport and dispersion. The objective of this report is to evaluate models for transport and dispersion and to make inferences about the basic underlying processes of fluvial hydraulics. The statement of this objective is necessarily general; the study is exploratory in nature. Hopefully the main results of the study will be firm indications of the directions in which future investigations should proceed.

Previous studies of transport and dispersion have measured only gross results such as a rate of transport or a concentration distribution. Usually the study was forced to resort to empirical relations to describe the data, or it attempted to derive the basic parameters from the results, unsatisfactory procedures in either case. The difficulties in extrapolating results from these procedures have pointed to the need to understand better the basic processes underlying transport and dispersion.

A specific objective of this study is the replication of Yang's experimental conditions as closely as possible in order to complete a set of dispersion data to include both the time development of the concentration distribution and the parameters of the step length and rest period distributions. This replication was achieved for Yang's dune flow condition. An attempt was made to replicate as well Yang's ripple flow condition but the transport-dispersion process proved extremely sensitive to small changes in flow conditions. Two ripple runs were completed such that information was gathered at slightly higher and slightly lower stream powers than Yang's ripple run. No attempt was made to match Yang's plane bed condition because he was unable to report dispersion data for that condition.

In addition to longitudinal dispersion the study should make inferences about the shape and movement of dunes, the vertical concentration distribution of tracers, the quantities of transport and the basic processes of bed feature formation.

The data collected in this study is limited in scope in that it was all collected in one flume with flow depths equal to approximately half a foot. Only two bed materials were used and the number of observations was, in some cases, less than is needed to establish firmly probability distributions.

On the other hand, large quantities of previous data have been collected in the same flume. In many cases the experimental conditions are similar to those in this present study. Good possibilities exist, therefore, for correlations.

The experimental data gathered consists of records of particle position over long periods of time, usually several days, for seven different flow conditions. Runs 1-3 are with 0.45 mm bed material and runs 4-7 are with 0.33 mm bed material. All pertinent hydraulic properties are reported as well as the properties of bed elevation as a function of distance and time. From the records of particle position, the lengths of the steps and the durations of rest periods are noted.

Theories of particle movement and dispersion are reviewed. Some of the possibilities arising from the knowledge of the basic quantities of step length and rest period are exploited. Inferences are made concerning the basic processes of transport and sand wave migration. Correlations between the statistical properties of step length and rest period and between flow properties are investigated. It is hoped that by establishing the most basic relations, perhaps the resultant sediment transport or dispersion could be evaluated as the interaction, or system, that it is.

Chapter II

MOTION OF SINGLE PARTICLES IN SEDIMENT TRANSPORT AND DISPERSION

A. Mechanism of particle movement

The behavior of a sediment particle under the influence of a single force field is deterministic. A single particle being transported over a movable bed is, however, subjected to diverse hydraulic forces so that its movement must be described as a random process.

Einstein (1937), in an early important treatment, used probability theory to describe the motion of single sediment grains. Using random walk techniques he was able to describe the particle motion as a sequence of rest periods and step lengths. For a continuous function to represent the discrete model he chose the exponential function. His calculations led to the expression

$$f_t(x) = e^{-(\sqrt{x} - \sqrt{t})^2} \cdot I_0(2\sqrt{xt}) \qquad (2\text{-}1)$$

where $f_t(x)$ is the probability density function at time t of the particle being at position x. $I_0(2\sqrt{xt})$ is a Bessel Function of the first kind of order zero. The quantity $f_t(x)$ also represents the dispersion distribution of a group of particles released at time zero.

Einstein (1950) again applied probability concepts to arrive at a function for calculating the bed load transport over an alluvial bed. Some significant assumptions that he made were as follows:

1. The probability of entrainment of a particle depends on the particle size, shape and weight, but not on its previous history.

2. The mean step length of a particle is a constant for the particle and is independent of the flow condition, the transport rate, and the bed composition.

The first of these assumptions is consistent with other theories of initiation of motion and reentrainment. Shields (1936) introduced a parameter for describing initiation of motion

$$\tau_* = \frac{\tau_c}{(\rho_s - \rho_f)\, g d_s} \tag{2-2}$$

where τ_* is Shields' dimensionless shear stress, τ_c is the bed shear at beginning of motion, ρ_s and ρ_f are the mass densities of sediment and fluid respectively, g is the acceleration of gravity and d_s is a representative grain si:-

Shields' parameter, which includes the particle size and weight, has been at least qualitatively verified by several investigators.

Gessler (1965) attached a probabilistic significance to the Shields' parameter. His work added probabilities to the Shields' curve of shear stress as a function of Reynolds Number. Shields' original curve corresponds somewhat to the 0.5 probability of Gessler's curve.

Liu (1957) added particle shape to the Shields' parameter as he introduced what he referred to as the "movability number" of the sediment particle,

$$\frac{U_*}{\omega} = f\left(\frac{U_*\, d_s}{\nu}, \text{particle shape factor}\right) \tag{2-3}$$

where U_*, the fluid friction velocity, is defined as

$$U_* = \sqrt{\tau/\rho_f}$$

where τ is the fluid shear stress at a particular point in the flow. The quantity ω is the fall velocity of the sediment particle and ν is the kinematic viscosity of the fluid. Liu considered the particle shape to be of secondary importance and eliminated it from equation (2-3). By retaining ω in the left hand side of the equation, however, Liu has retained the influence of shape factor. As seen in the U.S. Inter-Agency Report No. 12 (1957), particle fall velocity is clearly a function of the shape factor. It may be shown that Liu's parameter is essentially the square root of the Shields' parameter with the influence of shape added.

The second of Einstein's assumptions is open to question. It would appear that the mean step length of a particle is perhaps more related to the length of the bed forms than to the size of the particle. Goswami (1967) has shown that depending somewhat on the type of bed form, the lengths and amplitudes of bed forms are dependent on flow conditions and on bed composition. Mean step length must be related to the same quantities. One would expect that in addition to a relation between mean step length and mean bed form length, there would be relations between the higher moments of the probability distributions of the two quantities.

To properly consider the step length of a particle, one must examine the mode of particle movement. The usual classification of wash load and bed material load applies in this case. In this study, the wash load, or that part of the total sediment load which depends only on supply and not transport capacity, will not be considered. The bed material load is of interest. It generally moves in three modes of transport. The first of these, bed-load movement, implies that the particles move in almost continuous contact with the bed. The third, suspension, implies that the particles spend most of their travel time suspended by vertical turbulent velocity fluctuations. The second mode of transport, saltation, is intermediate between surface creep and suspension. Saltation implies a jumping mode of transport, and as originally introduced by Gilbert, depended on the impact of grains with each other (Bagnold, 1941).

Kalinske (1942) has shown that due to the impotence of the impact mechanism, saltation is not a significant factor in the movement of bed material load in water.

In this study of particle movement, it will be considered that suspension is the limiting case of bed-load movement. Grains move in a series of steps, each step being separated by a rest period of some duration. Depending on the intensity of the flow and the properties of the grains, the step lengths may be of different lengths and the rest periods of different durations. When a particle becomes suspended the step length becomes quite long in comparison to the step of a particle moving as bed load. At the other end of the spectrum of movement, when a grain just begins to stir at initiation of motion, the step lengths will be very short and the rest periods of quite long duration. In this study only those flow ranges where the particle step lengths and rest periods can be measured will be considered. Practically, this restricts the study to the ripple and dune regimes of flow. In plane bed flow there is a continuous exchange of particles between the bed and the fluid. The step lengths become very long and the rest periods very short. In fact, a particle may travel a long distance without experiencing a rest period. The particle may alternate between movement as bed load and suspended load.

The formation of bed features is directly related to the motion of individual particles in the bed. In fact, the entire process of flow over a movable boundary must be considered together. It is not feasible to separate the total process into isolated areas for study. For example, resistance to flow is related to the size of bed features which is related to flow intensity and sediment transport. The latter quantity is related intimately to the motion of individual particles.

The variables of loose-boundary hydraulics are subject to random fluctuations. The establishment of quantitative relations between the variables should be based on many observations. A complete discussion of these relations is beyond the scope of this report but selected observations are made.

B. Dispersion of sediments

The motion of individual particles is fundamental to the dispersion process.

Hubbell and Sayre (1964) developed a concentration distribution function for the dispersion process of natural sediments. Their assumptions were that the movement of particles could be represented as a process consisting of a series of alternating steps and rest periods. The length of steps and the duration of rest periods were considered to be independent of time or position such that the step lengths, X, and the rest periods, T, formed a set of independent, identically distributed random variables. Hubbell and Sayre assumed that the step lengths and rest periods were exponentially distributed with mean step length $1/k_1$ and mean rest period $1/k_2$ where k_1 and k_2 are the parameters of the exponential distribution represented by

$$f_Z(z) = \begin{cases} ke^{-kz} & z \geq 0 \\ 0 & \text{otherwise} . \end{cases} \tag{2-4}$$

By using these assumptions they arrived at the concentration distribution function

$$f_t(x) = k_1 e^{-(k_1 x + k_2 t)} I_0(\sqrt{2\, k_1 x\, k_2 t}) \tag{2-5}$$

where the symbols are as explained in equation (2-1) with the exception of the parameters k. Although Hubbell and Sayre used an entirely different technique from Einstein, they derived the same expression for $f_t(x)$ with the exception of the constants.

Hubbell and Sayre (1965) expanded their one-dimensional model into a general two-dimensional stochastic model with density functions unspecified. The derivation of the model (Sayre and Conover, 1967), which accounts for the vertical as well as longitudinal position of a particle at time t, proceeds from the joint distribution function

$$F_t(x,y) = \sum_{n=0}^{\infty} P\left[\sum_{i=0}^{N(t)} X_i \leq x,\ \sum_{i=0}^{N(t)} Y_i \leq y,\ N(t) = n\right] \tag{2-6}$$

where Y_i and X_i are random variables describing the vertical and longitudinal position of the particle and $N(t)$ is a counting function describing the number of steps a particle takes in time t. The distribution function, $F_t(x,y)$ refers to the probability that a particle has, at time t, travelled a distance equal to or less than x and is located at an elevation equal to or less than y (Sayre and Conover, 1967). To obtain the two-dimensional density function, equation (2-6) is differentiated

$$f_t(x,y) = \frac{\partial^2}{\partial x \partial y} F_t(x,y)$$

$$= \sum_{n=1}^{\infty} f_X^{(n)}(x)\, f_Y(y) \int_0^t f_T^{(n)}(t') \int_{t-t'}^{\infty} f_{T|Y}(\tau|y)\, d\tau\, dt' \ . \qquad (2\text{-}7)$$

Equation (2-7) is a rather formidable expression. For clarity, the assumptions necessary for its development will be stated and the terms will be explained individually.

The displacement process is assumed to be stationary with respect to time and homogeneous with respect to distance so that the random variables X_i and Y_i are independently and identically distributed with probability density functions $f_X(x)$ and $f_Y(y)$. The variable X represents the length of a single step and Y represents the elevation where the particle is located in the bed. The quantity n is the number of steps the particle has taken and is also equal to the number of rest periods it has had.

Further, it is assumed that the durations of rest periods and the elevations of successive depositions are independent. The duration of a rest period may, however, depend on the elevation at which the particle was deposited at the end of the previous step. The sum in equation (2-7) runs from $n = 1$ rather than $n = 0$ and

$$\int_0^{\infty} \int_{y_{\min}}^{y_{\max}} f_t(x,y)\, dy\, dx < 1 \ . \qquad (2\text{-}8)$$

In other words, the initial condition for the expression (2-7) is that the particle has moved once.

The expression $f_X^{(n)}(x)$ is the n-fold convolution of the density function $f_X(x)$ and is equal to the density function for the length of n successive steps. The expression $f_Y(y)$ is the probability density function for the elevation Y at which the particle is deposited. The expression $f_T^{(n)}(t')$ is the probability density function for the duration of n successive rest periods and is the n-fold convolution of $f_T(t)$ the density function for the length of a single rest period. The quantities t' and τ are dummy variables, defined as

$$t' = \sum_{i=1}^{n} T_i \tag{2-9}$$

and

$$\tau = T_{n+1} . \tag{2-10}$$

The significance of t' is that it is the duration of the first n rest periods and τ is the length of the $n + 1^{\text{st}}$ rest period.

The expression $f_{T|Y}(\tau|y)$ represents the probability density function for the length of rest periods given the elevation at which the particle is deposited. The density function for the duration of rest periods is related to this conditional density function by the expression

$$f_T(t) = \int_{y_{\min}}^{y_{\max}} f_{T|Y}(t|y)\, f_Y(y)\, dy . \tag{2-11}$$

Equation (2-7) may be integrated over either x or y to yield the marginal density functions according to the relation

$$f_{X_1}(x_1) = \int_{-\infty}^{\infty} f_{X_1,X_2}(x_1,x_2)\, dx_2 \;.$$

In the case of the longitudinal density function, this yields

$$f_t(x) = \sum_{n=1}^{\infty} f_X^{(n)}(x)\; P[N(t) = n] \;. \qquad (2\text{-}12)$$

If one assumes that $f_X(x)$ and $f_T(t)$ may be represented by the exponential distribution it follows that

$$f_t(x) = k_1 e^{-(k_1 x + k_2 t)} \sum_{n=1}^{\infty} \frac{(k_1 x)^{n-1}\,(k_2 t)^n}{\Gamma(n)\; n!} \qquad (2\text{-}13)$$

which is the same equation arrived at by Hubbell and Sayre in their earlier development.

Using the same procedure as Sayre in his general two-dimensional stochastic model, Yang (1968) developed a general one-dimensional model to describe the longitudinal dispersion process. Based on some measurements of step lengths and rest periods of plastic particles in a laboratory flume, Yang concluded that the exponential distribution represented well the rest periods and that the gamma distribution with two parameters represented well the step lengths. The exponential distribution is a special case of the gamma distribution with $r = 1$ which has a density function given by

$$f_Z(z) = \begin{cases} \dfrac{k^r}{\Gamma(r)}\, z^{r-1}\, e^{-kz} & z \geq 0 \\ 0 & \text{otherwise} \;. \end{cases} \qquad (2\text{-}14)$$

Using his assumed distributions and the general one-dimensional model, Yang derived the concentration distribution function

$$f_t(x) = k_1 e^{-(k_1 x + k_2 t)} \sum_{n=1}^{\infty} \frac{(k_1 x)^{nr-1}\,(k_2 t)^n}{\Gamma(nr)\; n!} \tag{2-15}$$

which for $r = 1$ reduces to equation (2-13).

To evaluate his model, Yang gathered experimental data on longitudinal dispersion. A detailed discussion of the data he gathered is included in Chapter 3.

In the same manner that the longitudinal concentration distribution function was derived as the marginal distribution so may the vertical distribution function be derived.

$$f_t(y) = \int_{-\infty}^{\infty} f_t(x,y)\; dx$$

$$= \sum_{n=1}^{\infty} f_Y(y) \int_0^t f_T^{(n)}(t') \int_{t-t'}^{\infty} f_{T|Y}(\tau|y)\; d\tau\; dt' \; . \tag{2-16}$$

It is of interest to examine some limiting cases of equation (2-16). Consider the integral

$$I = \int_{t-t'}^{\infty} f_{T|Y}(\tau|y)\; d\tau \; . \tag{2-17}$$

The significance of the integral is that it represents the complement of the probability that the length of the $n + 1^{st}$ rest period is equal to or less than $t - t'$. This probability is conditioned on the elevation of deposition at the end of the n^{th} step, the beginning of the $n + 1^{st}$ rest period.

The integration is carried out over the interval $[t - t', \infty]$, or over all possible lengths of the $n + 1^{st}$ rest period. Let $0 \leq t' < t$, then

$$I = F_{T|Y}(\tau|y)\Big|_{t-t'}^{\infty} = 1 - F_{T|Y}[(t-t')|y] \qquad (2\text{-}18)$$

when t becomes large, or when the duration of the $n + 1^{st}$ rest period goes to infinity,

$$I = \lim_{t\to\infty} [1 - F_{T|Y}\{(t-t')|y\}] = 0 \ . \qquad (2\text{-}19)$$

The quantity t is bounded by t' such that t cannot approach zero unless $t' = 0$. In any case where $t = t'$, or where the $n + 1^{st}$ rest period has zero duration,

$$I = 1 - F_{T|Y}(0|y) = 1 \ . \qquad (2\text{-}20)$$

By using equation (2-20) in equation (2-16) we get the result

$$f_t(y) = f_Y(y) \sum_{n=1}^{\infty} \int_0^t f_T^{(n)}(t')\, dt' \ . \qquad (2\text{-}21)$$

The integral in (2-21) is seen to be the distribution function,

$$\int_0^t f_T^{(n)}(t')\, dt' = F_T^{(n)}(t) \qquad (2\text{-}22)$$

for the length of n successive rest periods. For any time $0 < t = t' < \infty$

$$F_T^{(n)}(t) = 1$$

and

$$\sum_{n=1}^{\infty} F_T^{(n)}(t') \to \infty \ . \qquad (2\text{-}23)$$

By using (2-23) in (2-21) one sees that when $0 < t = t' < \infty$, $f_t(y)$ becomes infinitely large, indicating a singularity in equation (2-16) at that point.

When $t' < t < \infty$ equation (2-18) predicts that

$$I = \phi(t,y) < 1 . \tag{2-24}$$

In this event, equation (2-16) becomes

$$\begin{aligned} f_t(y) &= f_Y(y) \sum_{n=1}^{\infty} \int_0^t f_T^{(n)}(t')\ \phi(t,y)\ dt' \\ &= f_Y(y)\ \phi'(t,y) . \end{aligned} \tag{2-25}$$

Based on equation (2-25) it is evident that at any time t, the vertical concentration distribution is a function of the density function $f_Y(y)$ and the elevation of the previous deposition. The elevation at which the particle was deposited at the end of the n^{th} rest period is a logical determinant for $f_t(y)$ for the position of a single particle. When considering the vertical concentration distribution of many particles which have taken many steps, the parameter Y in the limiting case of equation (2-25) loses its obvious physical meaning. One expects intuitively for the case of many particles that

$$f_t(y) \rightarrow f_Y(y) . \tag{2-26}$$

Equation (2-7) requires for its correct evaluation a knowledge of the initial particle position (X_0,Y_0) or the initial distribution of many particles, $f_{t_0}(x,y)$. In the case of X, the initial position would be assumed to be station zero. In the case of Y, it is unknown what elevation to assume as a start. The distribution function in equation (2-6) had the information Y_0 contained in it. This information was lost in the process of differentiating to get equation (2-7). Since between rest periods the particle is vertically distributed according to the density function $f_Y(y)$, it is logical to assume the mean of the distribution $f_Y(y)$ as the starting point for the process (X_i, Y_i). This would tend to amplify the probability that the process Y_i would tend toward the distribution $f_Y(y)$ as implied by equation (2-26).

The only direct measurements of actual step lengths and rest periods were those by Yang (1968). His measurements indicated that rest periods are well represented by the exponential distribution and that step lengths are well represented by the gamma distribution. Yang's data is considered to be of rather limited quantitative value due to the rather artificial nature of his bed material. The form of his distributions should be essentially correct, however.

Crickmore and Lean (1962) used a transport model in which they assumed that every particle had an equal chance to move a certain step length. They concluded that a more detailed model taking into account the actual step length and rest period distributions was necessary.

C. Distribution of the step lengths and rest periods

At the present time there is no theoretical reason for the gamma distribution of the step lengths, or the exponential distribution of the rest periods. In fact, the gamma distribution is a flexible distribution which might be made to fit most skewed data with some degree of satisfaction. Since the exponential distribution is a gamma distribution with shape parameter $r = 1$, one can say that the rest periods are also gamma distributed. In fact, the gamma distribution with r slightly greater than one would probably be a more satisfactory theoretical model for the rest periods since a rest period of zero duration makes no sense and the maximum point of the exponential distribution is at time zero, as Yang used it. Empirically speaking, the exponential distribution is a simple and adequate representation of the gamma distribution with r near one.

The 2-parameter gamma distribution is equivalent to a Pearson Type 3 distribution with lower boundary zero. It is an attractively flexible distribution and its functional form is quite simple. Only two moments are required to estimate its parameters.

The mode of movement of a sediment particle assumed in the Hubbell-Sayre-Yang models is that a particle moves in a series of alternating steps and rest periods. A rest period is defined as the time between successive burials of the particle. A step length is defined, therefore, as the distance travelled between burials.

Physically it is possible for a particle to experience a negative step length. This can be handled with the gamma distribution using a simple transformation of coordinates. The negative step length would arise when a particle was swept back in the zone of reverse flow to be buried by the avalanching dune face.

If one observes sediment particles move, he sees that a particle comes at times to rest without being buried, being soon thereafter moved again. This is particularly true in plane bed flow. The definitions given above suffer, then, a slight loss in accuracy because the position of a particle can really be defined only when the particle is buried. The assumption is made that the time-duration of a step length is insignificant compared to the duration of a rest period. The gains achieved by this definition are large compared to the loss in accuracy suffered. One gain is a more tractable mathematical treatment. Another gain is the possibility of using some special experimental techniques to measure some of the quantities in equation (2-7).

Nordin (1968) gave a simplified model of the manner in which bed forms migrate. Figure 2-1 shows a series of two-dimensional bed forms moving in a downstream direction. The mechanism causing the downstream migration has been thoroughly discussed in the past, but never solved. Like many natural phenomena, it depends on many factors.

The model adopted herein is the same as that used by Sayre and Conover (1967) and discussed by Nordin (1968). No difference will be assumed in the manner in which grains move over ripple or dune beds. Bed form terminology is as defined by Simons and Richardson (1966). According to Simons and Richardson, there are differences between ripple and dune beds caused by changes in the basic mechanism forming ripples or dunes. The hypothesis is made herein that there is no difference in these bed forms. Hopefully, the experimental data will allow the testing of this hypothesis. It is acknowledged that the mechanism of movement of grains in upper regime flow (plane bed and higher) differs from that of the lower regime. The upper regime is beyond the scope of this study.

Figure 2-1(a) shows the flow pattern over a bed form. The fluid particles are subjected to an alternating sequence of expansions and contractions as they pass over the bed forms. The separation streamline extends from the crest of one dune to the back of the next. The separation zone, being a region of less stable flow, has the higher

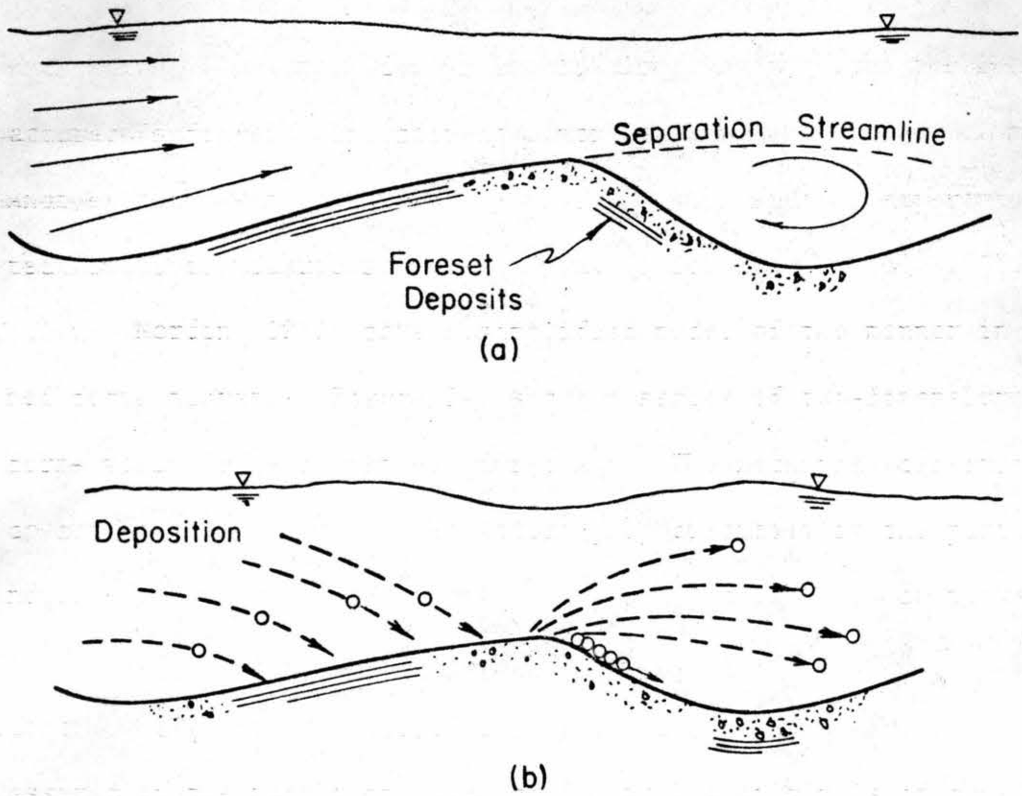


Figure 2-1. Typical bed form showing erosion and deposition.

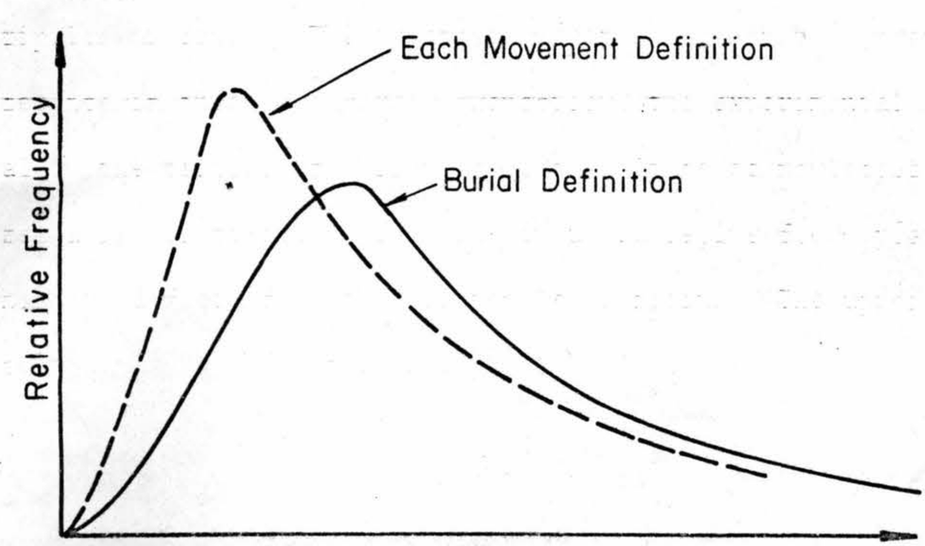


Figure 2-2. Change in frequency distribution with definition of variable.

intensities of turbulence. In figure 2-1(b) one sees a simplified description of particle movement. Grains are eroded from the back of the dune and begin to move. These grains join others which have been picked up from the trough by the turbulent velocity fluctuations and carried downstream. The majority of these grains avalanche down the face of the dune forming what geologists call "foreset deposits." These deposits are covered by more grains and the dune "marches" forward. The upstream angle depends somewhat on flow conditions whereas the downstream angle depends more on the angle of repose of the bed material. Of those particles which do not avalanche down the face of the dune, some are deposited in the separation zone and some travel greater distances. Depending on whether the grain reaches the back of the next dune, the step lengths vary in magnitude. A particle may be eroded from the back of one dune, be carried to the back of the next, avalanche down the face and be buried there. Another particle of identical properties might be eroded from the same dune and be carried much further depending on what conditions it was subjected to. One can clearly see that a frequency distribution rather than a mean value alone is required to define the movement of particles. Since most of the grains in a dune bed travel less than one dune length, one expects that the mean step length will be somewhat less than the mean dune length. Yang (1968) showed that dune lengths were gamma distributed. Perhaps some relation exists between all the moments of the step length and dune length distributions.

The moments of the probability distributions are defined by

$$m_a = \int_{-\infty}^{\infty} z_a \, f_Z(z) \, dz \qquad (2\text{-}27)$$

where m_a is the a^{th} moment of the distribution given by $f_Z(z)$, the probability density function of the random variable Z.

The significance of the burial definition, as opposed to the "each movement" definition of particle motion, becomes clear when considering the moments of the distributions. Figure 2-2 shows two frequency distributions for either step lengths or rest periods. Using the burial definition one gets the less skewed distribution, whereas the "each movement" definition results in an almost exponential distribution with the highest frequencies nearest the origin. One sees that most of the difference is near the origin so that moments about the origin are not drastically affected.

The form of the gamma distribution was given in equation (2-14). The two parameters r and k may be estimated from the mean and variance of a sample distribution from the relations

$$\bar{z} = \frac{r}{k} \qquad (2\text{-}28)$$

and

$$s_z^2 = \frac{r}{k^2} \qquad (2\text{-}29)$$

where $\bar{z}$ is the mean and s_z^2 is the variance. One sees from figure 2-3 how the shape of the distribution varies with the parameter r. Many types of skewed data can be made to fit one of these shapes.

From our definitions of step length and rest period, there are no admitted values of zero for either random variable; therefore the

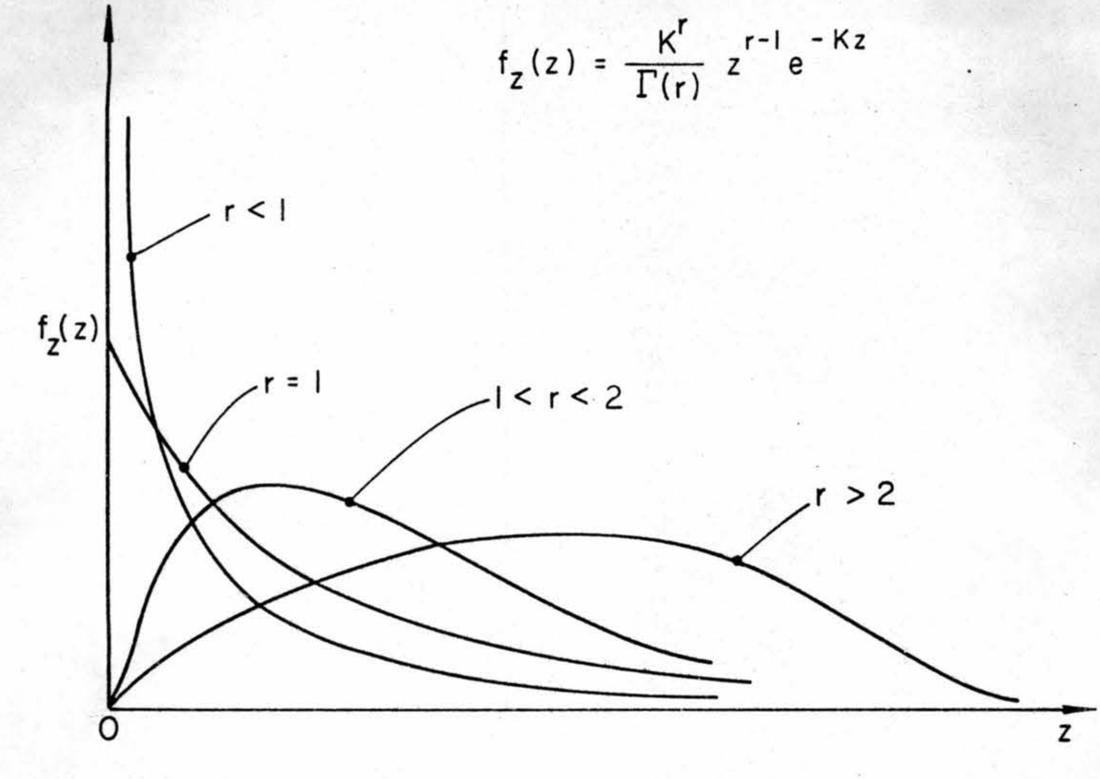


Figure 2-3. Effect of shape parameter on gamma distribution.

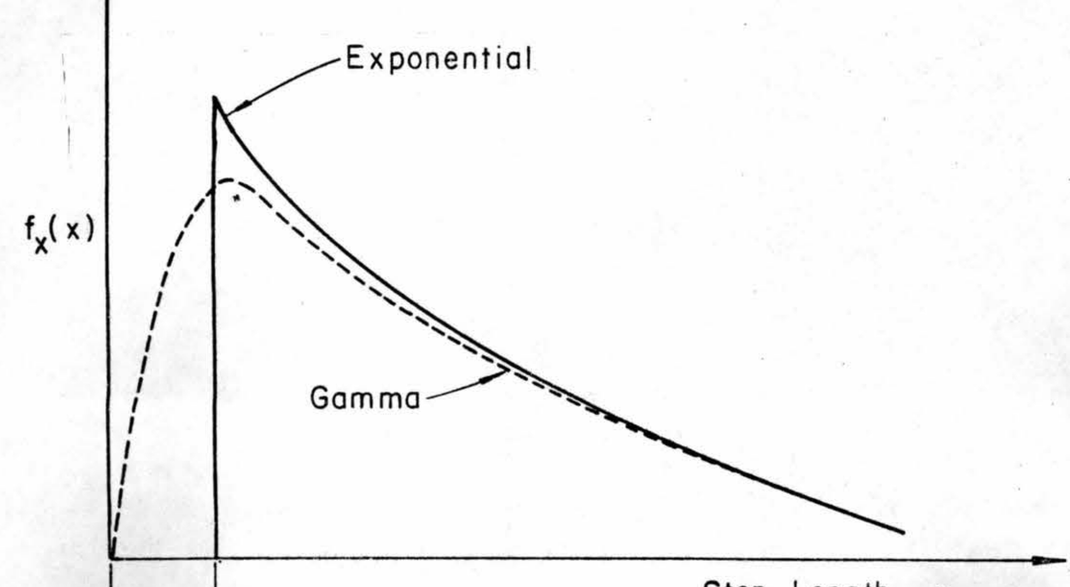


Figure 2-4. Similarity of exponential and gamma distributions.

value of r should be greater than one. In practice, r is estimated from moments and may assume any value greater than zero.

Recently, Todorovic and Shen (1968) have developed a stochastic model for the longitudinal motion of sediment particles. Their model, based on probability concepts treats the following random variables:

N_t - the number of steps in the time interval $(t_0, t]$.

t_n - the instant of time when the n^{th} step occurs.

X_n - the distance travelled in n steps from the starting point, X_0.

X_t - the distance travelled during the time interval $(t_0, t]$ from the starting point, X_0.

Z_n - the length of the n^{th} step.

Two cases are considered; the general case where the time of motion of the particle is considered and the approximation where a step is considered to occur instantaneously.

No solution for the model was possible in the general case. In the case where the time of flight was neglected they were able to solve for the density function of particle position after n steps,

$$f_n(x) = \frac{\lambda_2(x)}{\Gamma(n)} e^{-\int_{x_0}^{x} \lambda_2(s)\, ds} \left[\int_{x_0}^{x} \lambda_2(s)\, ds \right]^{n-1} \tag{2-30}$$

where $\lambda_2(x)$ is the probability that a particle moves in an interval Δx. The quantity

$$\int_{x_0}^{x} \lambda_2(s)\, ds$$

is equal to the expected number of stops in the interval $(x_0, x]$ and is related to the mean step length by the relation

$$\frac{x - x_0}{\int_{x_0}^{x} \lambda_2(s)\, ds} = \bar{X} = \frac{1}{k_1} \tag{2-31}$$

where k_1 is the parameter of an exponential density function. One sees then, if x_0 is taken as zero that

$$\int_0^x \lambda_2(s)\, ds = k_1 x \ . \tag{2-32}$$

Using (2-32) in (2-30) yields

$$f_n(x) = \frac{\lambda_2(x)}{\Gamma(n)} e^{-k_1 x} (k_1 x)^{n-1} \tag{2-33}$$

which, if $\lambda_2(x)$ = constant, becomes the n-fold convolution of an exponential probability density function.

Equation (2-33) depends for its value on the function $\lambda_2(x)$. Assuming

$$\lambda_2(x) = (k_1 x)^m \tag{2-34}$$

then (2-33) yields

$$f_n(x) = \frac{e^{-k_1 x} (k_1 x)^{n + m-1}}{\Gamma(n)} \ . \tag{2-35}$$

The distribution given by (2-35) is not of a known type but is similar to the convolution of a gamma distribution

$$f^{(n)}(x) = \frac{k_1}{\Gamma(nr)} (k_1 x)^{nr-1} e^{-k_1 x} \ . \tag{2-36}$$

Therefore some type of skewed distribution similar to a gamma might be deduced from (2-33) depending on the form of $\lambda_2(x)$. As Todorovic and Shen point out, however, from their equation (2-33) an exact gamma distribution with parameter r different from one is not possible.

The model of Todorovic and Shen says nothing of bed forms or turbulence. It is conceivable that an exponential distribution would fit the step lengths if the "each movement" rather than the burial definition were used. There would be a high frequency at the smaller step lengths as shown in figure 2-2.

Intuitively, the gamma distribution seems a better model than the exponential. Figure 2-4 shows how a gamma could approximate well the exponential distribution.

Using the burial definition of rest period one can relate the movement of bed forms to rest periods (Sayre and Conover, 1967). On figure 2-5 one sees that a record of $y(t)$, bed elevation as a function of time, provides a means of estimating rest periods conditioned on the elevations of deposition. The length of a rest period, given the elevation at which the particle is deposited, is equal to the duration of an upward excursion of the process $y(t)$ above the level of deposition. From a frequency analysis of the record $y(t)$, the frequency distributions leading to the conditional density function $f_{T|Y}(t|y)$ defined in equation (2-7) can be obtained.

D. Elevations of deposition

In order to complete the probabilistic description of the particle rest periods the probability density function for the elevations at which a particle is deposited, $f_Y(y)$ is needed. The model of particle movement previously presented allows particles to be deposited

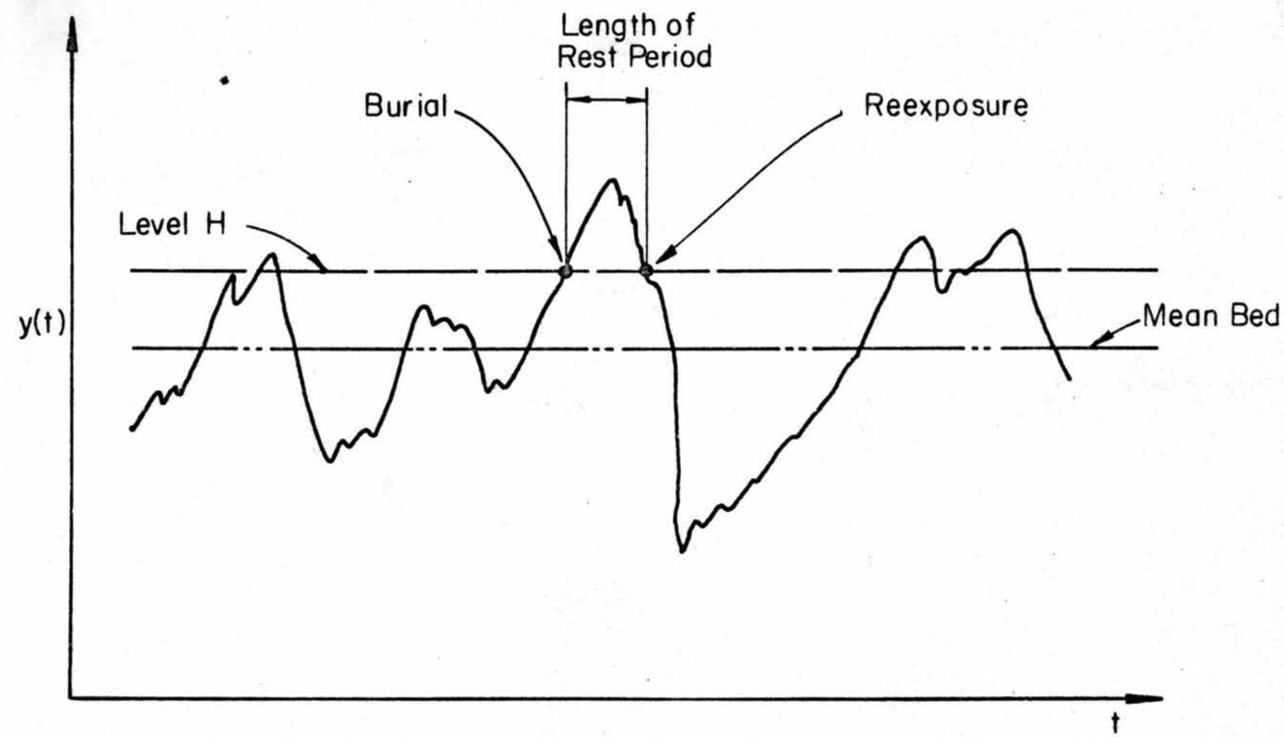


Figure 2-5. Bed form definition of conditional rest period.

only on the downstream faces of dunes. After deposition, the particle becomes buried and remains at that elevation until reexposed at a later time.

The bed forms move downstream as a result of the continuous erosion and deposition. For an equilibrium flow there can be no net scour or fill above or below the mean bed elevation. This means that for every particle that moves across the mean elevation another has to cross in the opposite direction.

Assume that all scour occurs in a layer of uniform thickness at a constant rate from the back of dunes. For the ideal bed form shown in figure 2-1 there should be the same amount of sediment scoured above and below the mean bed elevation; therefore, the same amount must be deposited above and below the mean bed elevation. The density function $f_Y(y)$ must have the same mean as the bed elevation for this case. If one further assumes that it is equally likely that a particle will be deposited at any elevation in the bed then the density function $f_Y(y)$ must be equal to the density function for bed elevation.

Several investigators have verified that bed elevation follows approximately a Gaussian distribution (Yang, 1968; Crickmore and Lean, 1962; Nordin, 1968). Yang has further shown that it matters little whether one considers all the bed elevation or only that portion of the elevation on the downstream faces of the dunes. The distribution remains nearly normal.

This ideal model of scour and deposition may not be entirely correct. It has not been rigorously tested as yet. The only condition required for an equilibrium bed is that the scour and fill balance at the different elevations. This condition is met if all of the scour and fill occurs in the troughs of dunes, say two standard deviations below the mean bed elevation. This would correspond to the quite possible situation where most of the particles avalanching down the faces of dunes settled at the base and most of the scour resulted from the turbulent eddies in the troughs. In this case $f_Y(y)$ would not be the same as the density function for bed elevation but would be skewed toward the lower elevations.

The limiting case of such a situation would be the case where no bed forms moved downstream, only up and down. Obviously this is not true since bed forms do march downstream. The model of Sayre and Conover (1967) must be partially correct to explain the movement of the dunes.

This suggested tendency for more particles to deposit at the lower elevations would not affect the longitudinal dispersion of sediments but would affect the vertical dispersion.

Yang (1968) found that smaller sediments dispersed faster than medium and coarser sediments. One explanation for this is that the smaller particles might have an affinity to be deposited in the higher elevations and might be moved more often than the coarser grains. In his core sample results, Yang found that the mean depth of penetration of tracer particles was usually below the mean bed elevation. There was considerable difference in his runs 1C and 1M; but otherwise there were no pronounced differences in the penetration of fine, medium or coarse particles.

Sayre and Hubbell (1965) also took core samples of tracer particles to determine the vertical concentration distributions. As with Yang's data, there were no pronounced or well defined distributions. There was, however, a tendency for the sediments to be concentrated at the lower elevations, suggesting that perhaps most of the deposition and erosion occurs at these elevations rather than higher.

Some insight on the deposition and erosion process may be gained from equation (2-11)

$$f_T(t) = \int_{y_{\min}}^{y_{\max}} f_{T|Y}(t|y)\, f_Y(y)\, dy \,. \tag{2-11}$$

Although it is not possible to solve (2-11) for $f_Y(y)$ some simple models may be tried for comparison with experimental results. The quantity $f_T(t)$ may be measured directly and $f_{T|Y}(t|y)$ may be estimated from bed form data.

It is convenient to use expected values to examine (2-11), thus

$$E(T) = \int_{y_{\min}}^{y_{\max}} E(T|Y)\, f_Y(y)\, dy \tag{2-37}$$

and

$$E(T^2) = \int_{y_{\min}}^{y_{\max}} E(T^2|Y)\, f_Y(y)\, dy \tag{2-38}$$

where $E(T^2)$ is related to the variance by

$$s_T^2 = E(T^2) - E^2(T) \,. \tag{2-39}$$

Equation (2-11) may be solved for $f_T(t)$ by assuming simple models for $f_Y(y)$. Two simple models are shown in figure 2-6 (P. W. Mielke, personal communication).

In case I,

$$f_Y(y) = \begin{cases} \frac{1}{4\sigma} & -2\sigma \leq y \leq +2\sigma \\ 0 & \text{otherwise} \end{cases} \quad (2\text{-}40)$$

and $f_{T|Y}(t|y)$ may be assumed to be a simple function of the form

$$f_{T|Y}(t|y) = b(y)\, e^{-tb(y)} \qquad 0 < t < \infty . \quad (2\text{-}41)$$

It is of interest to note the properties of this function. This may be done through an examination of the moments of the function. They are as follows:

$$\text{Mean} = {}_0m_1 = \frac{1}{b(y)} \quad (2\text{-}42)$$

$$\text{Variance} = {}_\mu m_2 = \frac{1}{[b(y)]^2} \quad (2\text{-}43)$$

$$\text{Skewness} = \frac{{}_\mu m_3}{\left({}_\mu m_2\right)^{3/2}} = 5 \quad (2\text{-}44)$$

$$\text{Kurtosis} = \frac{{}_\mu m_4}{\left({}_\mu m_2\right)^{2}} = 9 \quad (2\text{-}45)$$

where ${}_0m_1$ denotes the first moment about the origin and ${}_\mu m_i$ denotes the i^{th} moment about the mean.

One notes that the mean and variance are dependent on the bed elevation whereas the higher moment coefficients are not.

Let

$$b(y) = 2^{y/2\sigma} , \quad (2\text{-}46)$$

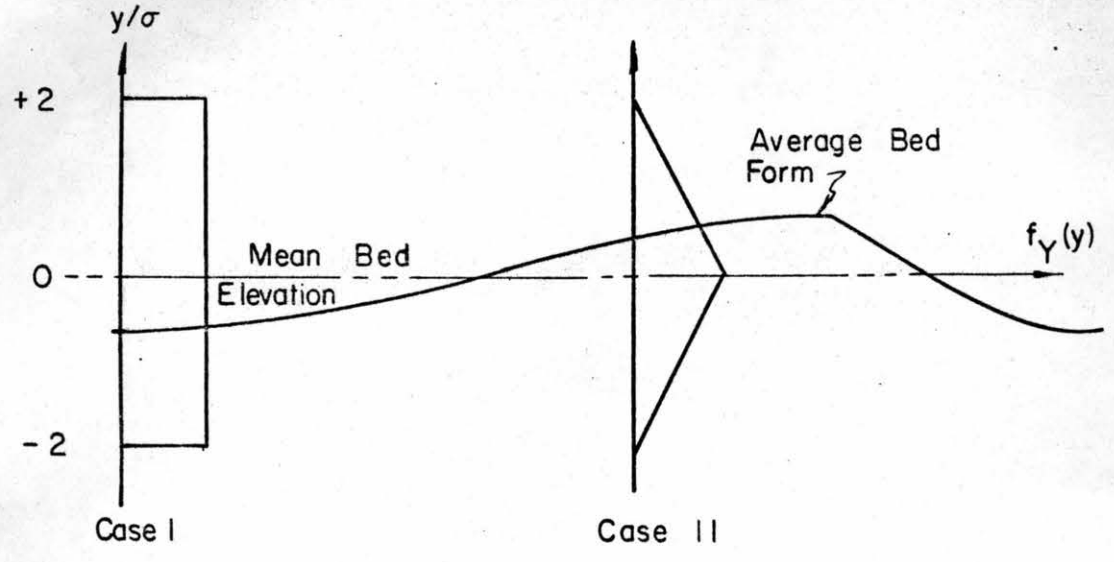


Figure 2-6. Models for $f_Y(y)$, the probability density function for the elevations at which particles are deposited.

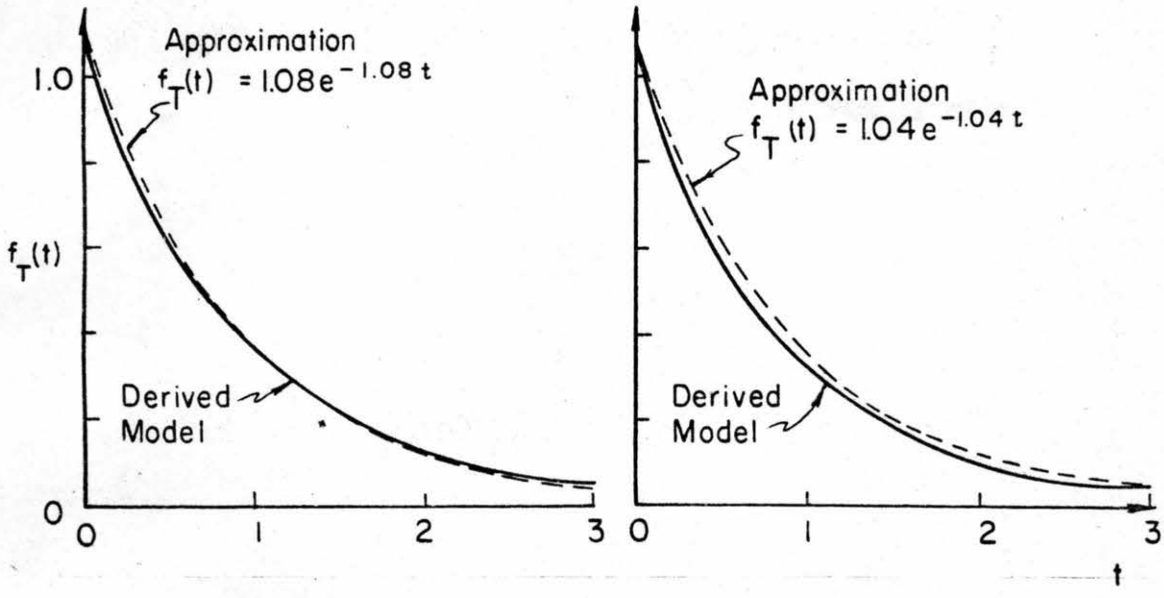


Figure 2-7. Models for $f_T(t)$ derived from conditional rest period density functions.

then

$$f_T(t) = \begin{cases} \dfrac{e^{-t/2} - e^{-2t}}{2t \ln 2} & t \geq 0 \\ 0 & \text{otherwise} . \end{cases} \quad (2\text{-}47)$$

As seen in figure 2-7, equation (2-38) can be closely approximated by the exponential function

$$f_T(t) = \begin{cases} 1.082\, e^{-1.082\, t} & t \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (2\text{-}48)$$

which is amenable to convolution and agrees with the exponential assumptions made by Yang.

In case II,

$$f_Y(y) = \begin{cases} \dfrac{2\sigma + y}{4\sigma^2}, & -2\sigma \leq y \leq 0 \\ \dfrac{2\sigma - y}{4\sigma^2}, & 0 \leq y \leq +2\sigma \\ 0 & \text{otherwise} . \end{cases} \quad (2\text{-}49)$$

Using the same functions for $f_{T|Y}(t|y)$ and $b(y)$ one gets

$$f_T(t) = \begin{cases} \dfrac{1}{t(\ln 2)^2}\left(\displaystyle\int_{1/2}^{1} e^{-ty} \frac{dy}{y} - \int_{1}^{2} e^{-ty} \frac{dy}{y} \right) & t \geq 0 \\ 0 & \text{otherwise} . \end{cases} \quad (2\text{-}50)$$

This function can as well be approximated by the exponential density function

$$f_T(t) = \begin{cases} 1.04\, e^{-1.04\, t} & t \geq 0 \\ 0 & \text{otherwise} . \end{cases} \quad (2\text{-}51)$$

The value of the assumed model for $f_{T|Y}(t|y)$ is to demonstrate the likelihood that $f_T(t)$ can be represented by an exponential distribution. The form of (2-11), however, allows different combinations of $f_{T|Y}(t|y)$ and $f_Y(y)$ to give the same $f_T(t)$. If $f_{T|Y}(t|y)$ and $f_T(t)$ are known, some insight of the form of $f_Y(y)$ can be inferred from experimental data.

E. Transport of sediments

Sediment transport is recognized as the result of many complex, interacting factors. In order to truly understand the effect of the different variables on the process, transport should be viewed as a random phenomenon. Using a combination of physical concepts and statistical parameters, one can develop models which apply in more general fashion than those models which approach the problem in simpler terms.

Crickmore and Lean (1962) and later, Sayre and Hubbell (1965) used statistical quantities to derive formulae for bed material being transported as bed load. Assuming that particles move in a layer of uniform thickness which can be represented by a single dimension, δ, the rate of transport is given by Sayre and Hubbell (1965) as

$$q_s = \gamma_s(1 - \lambda)\ \delta\ \frac{\bar{x}}{t} \tag{2-52}$$

where q_s is the transport in weight per unit time per unit width, γ_s is the specific weight of the sediment, λ is the porosity of the bed material, and $\bar{x}$ is the location of the mean of the concentration distribution at any time t. To use this model requires a knowledge of the quantities δ, $\bar{x}$ and t which are obtainable from tracer experiments.

Using the assumption that both the step lengths and rest periods are exponentially distributed, Sayre and Hubbell (1965) determined that

$$\frac{\bar{x}}{t} = \frac{k_2}{k_1} \quad . \tag{2-53}$$

If one assumes the gamma distribution for step lengths, then

$$\frac{\bar{x}}{t} = \frac{1}{t} \int_0^\infty x \; f_t(x) \; dx = \frac{k_2}{k_1} \, r \tag{2-54}$$

and

$$q_s = \gamma_s (1 - \lambda) \; \delta \, \frac{k_2}{k_1} \, r \quad . \tag{2-55}$$

Chapter III

EXPERIMENTAL EQUIPMENT AND PROCEDURE

The objective of the experimental program was to obtain a set of unified data of hydraulic and sand bed properties along with the step lengths and rest periods of individual particles.

It is attempted to present herein a rather complete set of the data collected. Included in the Appendix is the set of step length and rest period data collected. Properties of the tracer particles and bed materials are given in this section along with the hydraulic conditions prevailing during the individual runs. Some of the statistical properties of the bed forms are given. The complete bed elevation records are on file elsewhere.

It was desirable to match the experimental conditions of Yang (1968) so that the actual values of step lengths and rest periods could be tested against his dispersion data. As discussed previously, this has been done with some success. It was difficult to replicate exactly Yang's ripple run where the slope was 0.00088 feet per foot. It was found that a small change in water surface slope had a profound effect on the movement of bed forms. Data was taken for two ripple conditions which bracket Yang's ripple run. Also, Yang's run 2, dunes, was matched quite well. It was not attempted to match his plane bed run since the step lengths should be quite long and measurements would have had little meaning.

Since many comparisons are made between the results of this present study and the results obtained by Yang, a short discussion of the data obtained by Yang is in order.

Yang's objective was to measure longitudinal dispersion under ripple, dune and plane bed conditions. A partial summary of Yang's data is given in table 3-1.

His runs are identified by number, representing the bed form as follows:

Run 1	Ripples
Run 2	Dunes
Run 3	Plane bed

and by size of tracer as follows:

F	Fine tracer	(0.18-0.21 mm)
M	Medium tracer	(0.30-0.35 mm)
C	Coarse tracer	(0.50-0.59 mm)

Thus Yang's run 2M was a dune run with medium sized tracers.

Yang's bed material had a median size of 0.33 mm. Its size gradation is shown in figure 3-1.

Some data presented by Nordin (1968) was used to compare with data collected in this present study. Nordin's data is concerned with the statistical properties of sand waves. His data, along with similar data collected in this study, are presented in table 3-2. Some of Nordin's data was derived from a computer analysis of Yang's data. In this case, Nordin is listed as the reporter and Yang as the source.

It is necessary to compare data collected in this study with data collected by Yang, Nordin and others. Every effort is made to avoid confusion as to whose data is being discussed. To clarify matters a short discussion of the data used is in order. Four sources of data

TABLE 3-1. SUMMARY OF YANG'S DATA

Run Number	1F	1M	1C	2M	2C
Water surface, slope x 10^2	0.088	0.088	0.088	0.212	0.204
Normal depth, ft	.522	.518	.499	.521	.555
Water discharge, cfs	1.14	1.14	1.14	1.69	1.70
Total sediment concentration, ppm	82.2	60.2	88	872	615
Velocity of tracer, ft/hr	1.13	.585	.848	4.7	4.1
Rate of spreading of tracer, ft^2/hr	6.48	1.72	2.68	20.2	16.6
Size of tracers, mm	0.18-0.21	0.30-0.35	0.50-0.59	0.30-0.35	0.50-0.59
Bed form	Ripples	Ripples	Ripples	Dunes	Dunes

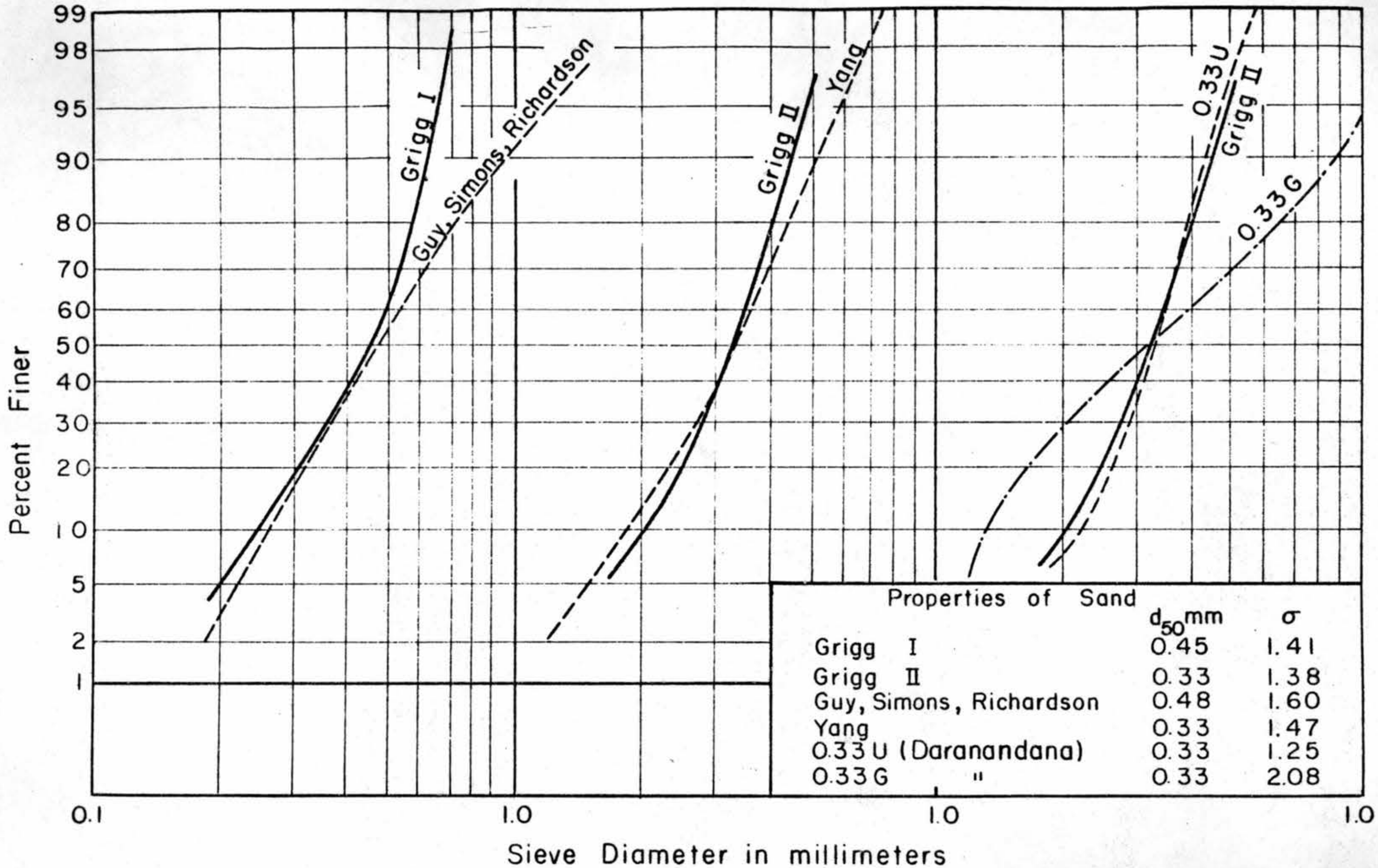


Figure 3-1. Size distributions of bed materials.

TABLE 3-2. SUMMARY OF BED FORM DATA

Reporter	Run No.	Slope	V fps	D ft	d_{50} mm	σ_y ft	$\bar{L}_x$ ft	$\bar{L}_x / \bar{L}_t$* ft/min	Source of Data	Bed Form
Grigg	1	0.00088	1.12	0.521	0.45	0.0394	1.43	0.0303	2-ft flume	Ripples
	2	.00223	1.55	.462	.45	.0536	2.82	.0913		Dunes
	3	.00440	1.97	.570	.45	.0730	3.75	.334		Dunes
	4	.00119	1.14	.511	.33	.0374	1.30	.0563		Ripples
	5	.00069	0.97	.595	.33	.0354	1.08	.0158		Ripples
	6	.00212	1.59	.543	.33	.0590	2.75	.131		Dunes
	7	.00156	1.51	.523	.33	.0442	2.39	.131		Dunes
Nordin	17,19	.00063	2.09	2.80	.24	.169	4.81	.288	8-ft flume	Dunes
	32,33	.00056	2.01	2.36	.24	.127	4.23	.288		Dunes
	40,43	.00088	1.10	.518	.33	.0432	1.20	.0952	2-ft flume	Ripples
	44,46	.00088	1.07	.522	.33	.0390	1.29	.0368	(Yang)	Ripples
	48,49	.00212	1.62	.521	.33	.0700	2.88	.100		Dunes
Nordin	1-3	.00056	2.12	2.26	.23	.268	5.83	-----	Atrisco Lateral	Dunes
Nordin and Algert	4	.00136	1.91	.670	.28	.115	4.73	-----	8-ft flume	Dunes
	5	.00134	2.11	1.05	.28	.175	8.63	-----		Dunes
	6	.00058	3.62	2.60	.23	.647	24.6	-----	Bernardo	Dunes
	7	.00058	2.48	4.15	.23	.745	25.4	-----		Dunes

*$\bar{L}_x$ and $\bar{L}_t$ are the mean distances between zero crossings of the $y(x)$ and $y(t)$ bed form data respectively.

are used: The principal data collected for this study by GRIGG, the dispersion data collected by YANG (1968), the sand wave data reported by NORDIN (1968) and sediment data reported by GUY, SIMONS AND RICHARDSON (1966). The latter data includes that initially reported by Daranandana (1962). The data collected by Grigg and Yang is compared often in this study. Grigg's runs are referred to by number, as 1 through 7. Yang's runs are referred to by number and letter. Yang's runs 1C, 1M and 1F compare to Grigg's runs 4 and 5; his runs 2C and 2M compare to Grigg's run 6. Data reported by Nordin and by Guy, et al. are used at isolated points and are identified clearly.

The following discussion describes the experiments conducted by Grigg, except where otherwise noted.

A. Flume

The experimental flume was of the recirculating type, 60 feet long by 2 feet wide. The side walls were made of 1/2-inch plexiglass so that the movement of bed forms could be observed. The slope of the flume could be adjusted from 0 to 10 percent and the discharge from 0 to 8 cfs. This flume has been used on several sand bed investigations at Colorado State University. It has been further described with schematic drawings and pictures by Yang (1968).

The flume is serviced by an instrument carriage which traverses either manually or automatically the length of the flume. The speed of the carriage may be varied to match experimental conditions. The position of the carriage is monitored by an event marking mechanism which closes a microswitch at each one-foot station.

B. Sand

Two bed material sands were used in the experiments. The first bed material, used in runs 1, 2 and 3 was of median sieve diameter 0.45 mm. This size was chosen to match the size of tracers obtainable. The second bed material, used in runs 4-7, was of median sieve diameter 0.33 mm to match Yang's sand. Approximately 3 tons of sand were used in each case.

The sands in this present study were purchased from the Ottawa Silica Company through their distributor, Van Waters and Rogers in Denver. The desired size distributions were obtained by mixing different types of Ottawa sands.

Figure 3-1 shows the size distribution of both sands along with some previously tested sands for comparison. All sizes given are sieve sizes. Where other sands were originally presented as fall diameter distributions, they have been converted to sieve size distributions through the use of data in Report No. 12 (U.S. Inter-Agency Committee on Water Resources, 1957).

Shown for comparison with the 0.45 mm sand is the 0.45 mm sand from Guy, Simons and Richardson (1966). After converting to sieve sizes this sand becomes a 0.48 mm sand. It should have some comparative value, however, when viewing hydraulic properties and sediment transport.

It is seen that the 0.33 mm sand matches fairly well Yang's sand. The small differences are attributed to the fact that Yang's sand was a river sand, sieved between two sieves whereas the present sand was a mixture of different Ottawa sands. It should be noted that sieving large quantities of river sand such as was done by Yang is a time consuming and expensive process. The Ottawa sands can be purchased economically in bags and mixed rapidly. In addition, it should be easier to reproduce bed materials formed of the Ottawa sands. The Ottawa sands are slightly more rounded than the natural river sand used by Yang.

The 0.33 U and 0.33 G sands shown are uniform and graded sands respectively, extensive tests of which are reported by Daranandana (1962) and summarized in Guy, et al. (1966). As seen on figure 3-1, these sands have approximately the same median size as the present sand and bracket it in gradation. The 0.33 U sand is actually quite close to the 0.33 mm sand used in this study. The measure of gradation, σ, is defined by

$$\sigma = \frac{1}{2}\left(\frac{d_{84}}{d_{50}} + \frac{d_{50}}{d_{16}}\right) \tag{3-1}$$

where d_i is the size for which i percent of the sediment sample is finer. This measure of gradation is the standard deviation of a log normal frequency distribution and is sometimes called the geometric standard deviation.

There is much to be said for using well graded river sands in flume experiments. The gradation has an effect on resistance to flow (Simons and Richardson, 1966) and median diameter (d_{50}) alone is not sufficient to fully describe a bed material. It was considered more important in this study to match Yang's sand than to point toward field conditions with a graded sand. After the dispersion and transport process is better understood for uniform sands the effects of gradation can be better studied.

C. Tracer particles

The tracer particles used in the study were radioactive microspheres developed by the Nuclear Products Division, 3M Company for medical applications. Modifications were made to the ordinary 3M specifications in order to approximate more closely the properties of sediment particles.

The tracers were made of ceramic material nearly spherical in shape (Lahr, Grotenhuis and Ryan, 1963). The ceramic material is able to absorb and retain a radiosotope. The radioisotope chosen for this application was Scandium 46, of selected activity and short (84.0 days) half-life. It was felt that the short half-life would minimize the hazard associated with losing a particle. Prior to the experiments the likelihood of losing one or more of the particles was assumed to be high. No particles were lost, however, even after some 2000 particle-hours of use, much of which included recirculation of the particles through the flume return pipe.

The ceramic microspheres had an initial mass density of approximately 3.0 gm/cc. After treating with the Scandium isotope the specific gravity was considerably reduced. In order to attain the desired value of 2.65 gm/cc, a nickel coating was applied to increase the density. The nickel had to be applied prior to irradiation, the quantity being selected by a very tedious and expensive trial and error process. The nickel became very fragile in some cases and broke off with very little handling, especially with the higher activities. It was very difficult to obtain exact densities of 2.65 gm/cc; therefore the particles had to be used with densities of from 2.45 to 2.64 gm/cc.

Table 3-3 lists all of the pertinent properties of the particles as they were measured upon receipt. The activity could not be directly measured on an absolute basis due to limited equipment, but could be verified on a relative basis. The specific gravities were calculated using measured diameters and fall velocities. The relation used was the drag coefficient-Reynolds Number relation given in Report No. 12 (U.S. Inter-Agency Committee on Water Resources, 1957).

It was found that more damage occurred to the particles through handling than in the flume; therefore the particles were allowed to remain in the flume from one run to the next when possible. On a ripple run, the mean velocity of the particles was too slow to allow this, so the particles were relayed to the upper end of the flume after they passed station 50. Only the reach between stations 10 and 50 was used in order to eliminate the effects of entrance and exit conditions. The first step length or rest period observed was always discarded when it was suspected that the observation was not under equilibrium conditions.

There was a wearing effect of the nickel coating on the particles. The effect of this abrasion was to decrease the specific gravity of the particles. Figure 3-2 shows an approximate relation between hours run and percent change in specific gravity. From this relation one can conclude that the abrasive effect is present and limits the useful life of the particle. In addition, the nickel shells will break after too much use as noted in table 3-4. The breakage always occurred when the particles were handled with tweezers, creating a condition of extremely high shell stress. The particles tended

TABLE 3-3. PROPERTIES OF TRACER PARTICLES BEFORE TESTS

Particle Number	Activity (microcuries)	Base Date	Fall Diameter mm	Standard Fall Velocity cm/sec	Diameter mm	Specific Gravity
1	12.5	8-20-68	0.46	7.2	0.45	2.70
2	13.4	8-20-68	.42	6.5	.45	2.50
3	14.0	9-13-68	.34	5.1	.37	2.45
4	14.4	9-13-68	.38	5.8	.41	2.43
5	15.2	9-13-68	.32	4.7	.36	2.36
6	41.9	9-13-68	.32	4.7	.33	2.56
7	43.6	9-13-68	.32	4.7	.32	2.64
8	41.0	9-13-68	.35	5.3	.32	2.88
9	94.0	9-24-68	.28	4.0	.44	1.81
10	94.0	9-24-68	.30	4.4	.46	1.85
11	90.0	9-24-68	.32	4.7	.44	1.98

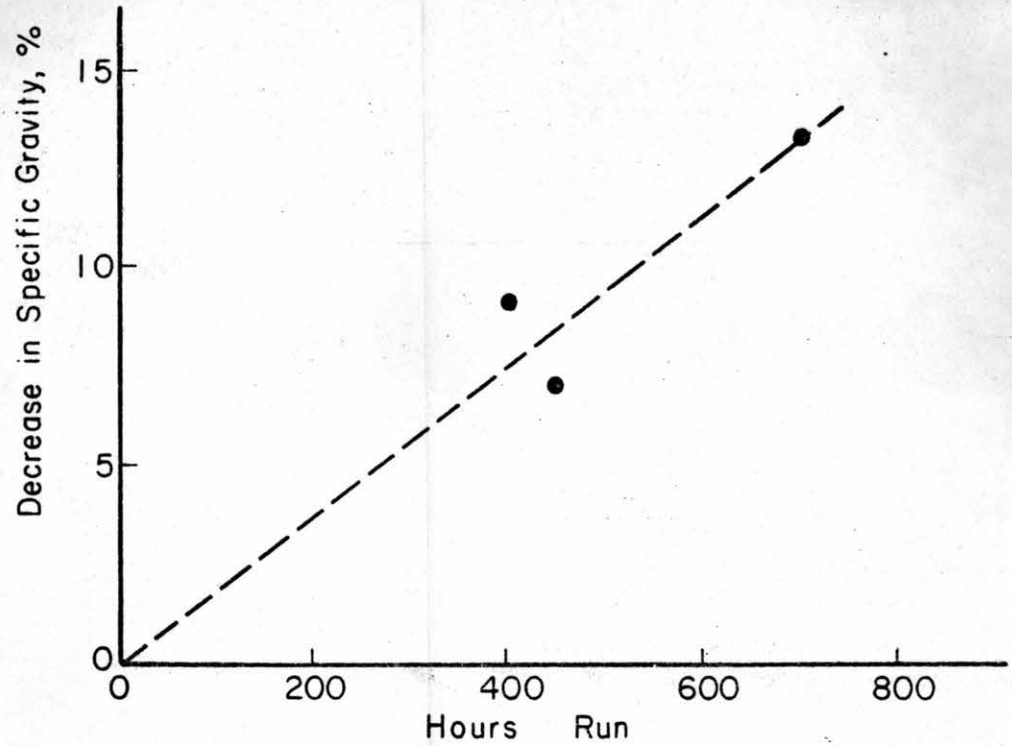


Figure 3-2. Change in particle specific gravity with time in flume.

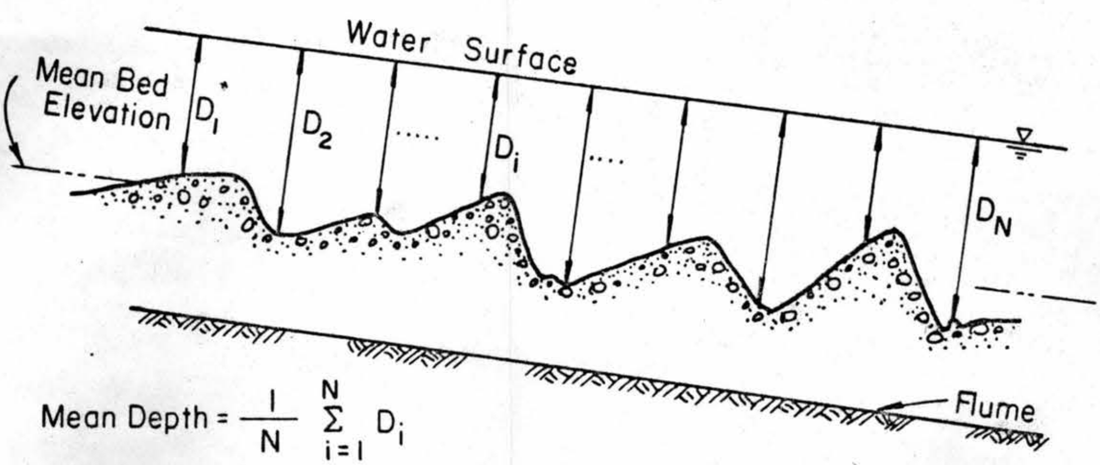


Figure 3-3. Definition of mean depth of flow.

TABLE 3-4. PROPERTIES OF TRACER PARTICLES AFTER TESTS

Particle Number	Fall Diameter mm	Standard Fall Velocity cm/sec	Diameter mm	Specific Gravity	Percent Change in Specific Gravity	Hours Run	Comments
2	0.38	5.8	0.45	2.27	9.2	400	
3	.32	4.7	.37	2.29	7.0	450	
6	---	---	.33	----	---	450	Broke on handling after run no. 7
7	.28	4.0	.32	2.33	13.3	700	Broke on handling after test

Note: Other particles not used extensively in flume tests.

to break most easily if they had been used extensively and/or if they had high activity. Particle number 2 never broke even after extensive use and handling. The activity of particle 2 was only 13.4 microcuries.

The initial plan was to use all identical 0.33 mm particles and to distinguish between them by activity. Due to the difficulty in obtaining 0.33 mm particles, initial experiments were begun with the 0.45 mm particles. When the 0.33 mm particles became available the bed material was changed. Three particles were then used: numbers 3, 6 and 7. One sees from table 3-3 that these particles had fall diameters of 0.34, 0.32 and 0.32 mm respectively. Their specific gravities ranged from 2.45 to 2.64 before the experiments. It was not possible to use the higher activity particles (greater than 90 microcuries) due to their low specific gravity and fragile condition. Therefore, it was not possible to truly distinguish particles on the basis of activity. By carefully following the particles, however, it was possible to keep track of them within reason. It was assumed that there was no difference between particles 6 and 7 so that it was not necessary to distinguish them other than to measure an individual step length or rest period.

D. Hydraulic conditions

The hydraulic properties of interest are given in table 3-5. Runs 4 and 5 bracket Yang's Run 1 while Run 6 is practically the same as Yang's Run 2.

In all runs the desired slope and discharge were determined prior to starting the flume. The criteria for determining these parameters were the matching of Yang's runs and the covering of the

TABLE 3-5. HYDRAULIC CONDITIONS DURING EXPERIMENTS

Run No.	Q, cfs	Slope	Depth, ft	Temp., °C	d_{50} mm	Dates	Bed Form	Particle No.	ppm
1	1.14	0.00088	0.521	20.0	0.45	9-03-68 to 9-12-68	Ripple	2	---
2	1.40	.00223	.462	20.0	.45	9-16-68 to 9-25-68	Dune	2 7	182
3	2.20	.00440	.570	20.0	.45	9-29-68 to 10-03-68	Dune	2 7	1041
4	1.14	.00119	.511	20.0	.33	10-08-68 to 10-18-68	Ripple	3 6 7	28
5	1.12	.00069	.595	20.0	.33	10-22-68 to 10-24-68	Ripple	3 6 7	---
6	1.69	.00212	.543	19.0	.33	10-27-68 to 11-02-68	Dune	3 6 7	450
7	1.55	.00156	.523	19.0	.33	11-06-68 to 11-13-68	Dune	3 6 7	350

ripple-dune flow regime. With the 0.45 sand, the slope was varied from 0.00088 to 0.00440. In all cases the depth was held near half a foot. In the second series of runs, with the 0.33 sand, Yang's conditions 1 and 2 were first studied; then it was attempted to increase the slope past the 0.00212 value. No greater slope could be obtained, however, without a plane bed forming. This effect was similar to that observed by Daranandana (1962) with his uniform sand. There is quite an abrupt change from dunes to plane bed with the uniform 0.33 mm sand. This observation confirms the results obtained by Daranandana but with a different bed material.

After setting the slope and discharge to the desired values, the water surface was brought to a level parallel with the flume slope by the use of the tailgate. After uniform flow conditions were established with the water surface, observations were begun of the mean bed level of the sand bed. Continuous observations were made of the water surface and sand bed profiles using a point gage. It was observed that each flow condition had a unique amount of sand in movement, leaving a certain amount as bed material in the flume. The bed would aggrade or degrade until the proper elevation was reached. Only then would equilibrium conditions hold. For ripple conditions it required a week or ten days to reach equilibrium depending on the bed elevation one started with. Dune runs required less time, usually about three days. After equilibrium conditions were reached, the flume would run continuously for days with little variation in water surface or mean bed elevation.

The discharge was read from a manometer connected to an in-line orifice meter. During one run the calibration of the meter was given an approximate test using an Ott current meter.

The basis for measuring the water surface slope was a careful survey of the flume slope. The water surface elevation was then related to the flume elevation by point gage readings taken every two feet along the flume.

During each run the temperature of the water was maintained as close to 20 degrees centigrade as possible. The temperature could be lowered by adding cold water into the tailbox.

The mean bed elevation and depth of flow were obtained using point gage readings as shown in figure 3-3. Several series of point gage readings were made over the period of each experiment and the mean values were calculated.

E. Radiation detection equipment

The radiation detection equipment used was essentially the same as that used by Yang (1968). The use of the equipment was slightly different, however, in that only the location and intensity of radiation were of interest rather than the distribution of intensity.

Figure 3-4 shows the general connection of the radiation equipment. As the scintillation detector passes over the particle it senses the radiation. The lead collimator filters and sharpens the signal to aid in the location of the maximum intensity. The detector sends a voltage to the radiation analyzer which may be set to distinguish between voltage pulses if desired. In this experiment the radiation analyzer was used only to regulate voltages, notably the input high

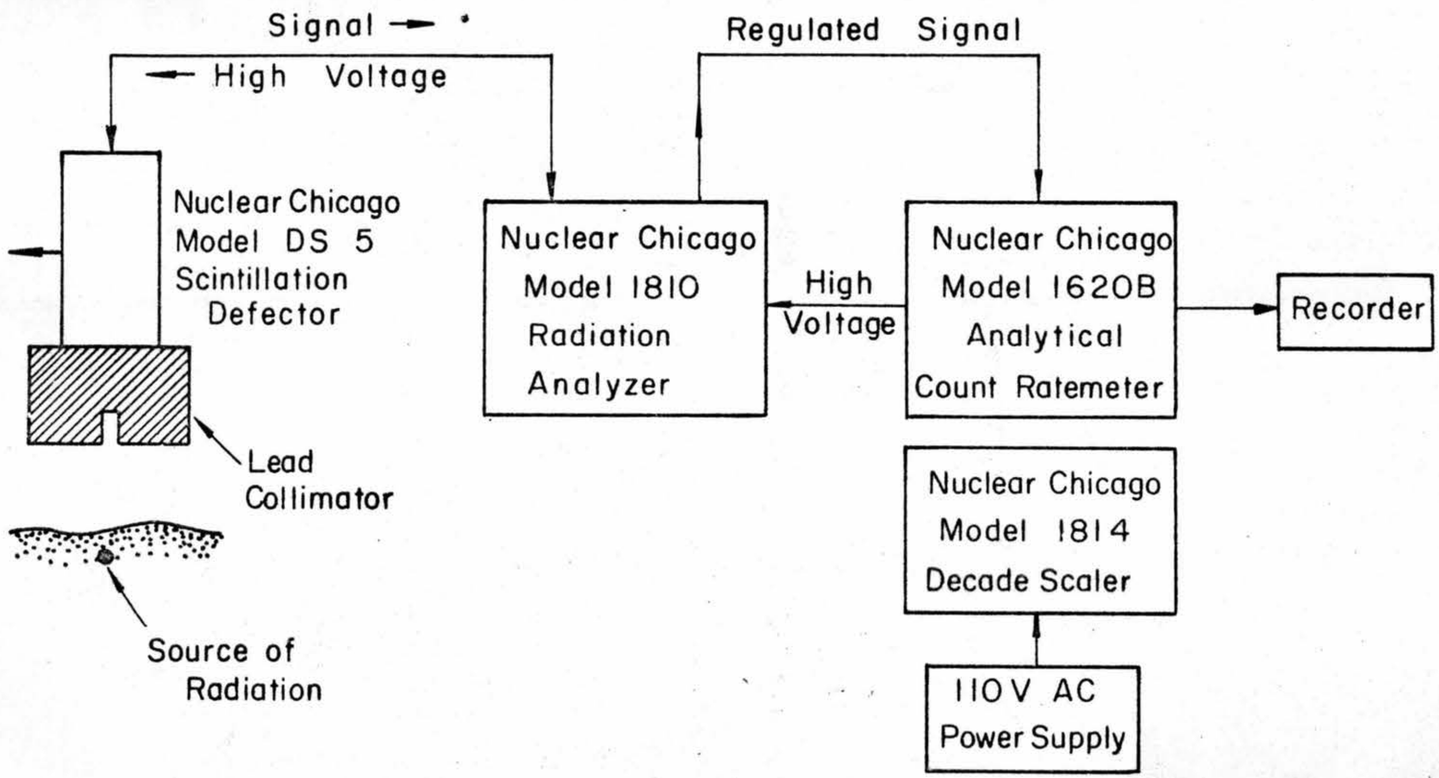


Figure 3-4. Functional diagram for radiation detection equipment.

voltage from the ratemeter. The voltage pulse from the analyzer is fed either to the ratemeter or to the scaler, as desired. The ratemeter provides a means to average the high frequency pulses received and to adjust the sensitivity of detection. The scaler provides a means to make an accurate count of the pulses received.

In this experiment the ratemeter was set to deliver an averaged voltage to a Mosely Model 680 Strip Chart Recorder. The recorder then produced a time record of sensings of the radioactive particles. The longitudinal position of the particles was measured with an estimated accuracy of one-tenth of a foot.

Due to the necessarily long duration of the individual runs, it was necessary to let the carriage traverse unattended. The cycling period of the carriage was long enough (approximately 11.75 feet per minute speed or 3.4 minutes cycle time) to cause appreciable error in the measurement of a rest period. Further, if step lengths occurred very frequently, one might have been missed in the traverse time. The average error in measuring rest periods was the cycle time or 3.4 minutes. The maximum error was twice the cycle time or 6.8 minutes. As the measured rest periods ranged from 9.7 to 124 minutes, the average error ranged from approximately 4 percent to 35 percent with the usual case being on the order of 15 percent.

F. Sonic sounding equipment

Two records of bed elevation were made for each run. A stationary transducer was mounted to sense the record $y(t)$ or the bed elevation at a fixed point over a period of time. A moving transducer was fixed to the instrument carriage and a periodic record of $y(x)$ was made. For the $y(x)$ soundings it was considered that the record was instantaneous. A $y(x)$ sounding was made at about time intervals corresponding to the time it took for a bed form to migrate from one end of the flume to the other.

The transducers fed a Automation Industries Inc. Dual Channel Stream Monitor, Model 1042, similar equipment to that used in many previous investigations at Colorado State University. One channel was used for the $y(t)$ record and the other for the $y(x)$ record. Both outputs were recorded on Mosely Model 680 Strip Chart Recorders.

The continuous analog records of bed elevation were converted to digital records on an Auto-trol Corporation analog-to-digital converter. The output of the converter was to computer cards such that all statistics could be directly computed on the computer.

G. Total bed material transport

A total load sampler was mounted on the end of the flume enabling samples to be taken of all the sediment moving. The sampler, identical to that used by Yang (1968) sampled the entire thickness of the nappe and allowed lateral integration of the samples. The samples were run into a calibrated volumetric tank where the total volume of the water sediment mixture could be directly measured using a point gage. The concentration of sediment could then be determined by drying and weighing each sample. Usually about 20 samples were taken during each run to obtain a representative average. Practically no material was moving in suspension during any of the runs so that the bed load transport was essentially the quantity being measured.

H. Determination of vertical concentration distribution

A quantity of interest was the effect of particle size on vertical concentration distribution. Accordingly, three sizes of fluorescent tracer particles were prepared from the bed material. Approximately 550 grams of each of the three sizes were prepared. This was enough tracer to ensure detectable quantities in each core sample. The sizes and colors selected were as follows:

Sieve Class	Color
0.177 - 0.250 mm	Orange
.250 - .350	Green
.350 - .500	Blue

The colors were selected based on past experience for ease in detection.

During the early part of Run 7, the total mixed sample of the three sizes was injected as a point source at the water surface at station 5. The particles were allowed to disperse all during the course of the run, a time of approximately 5 days. After the run was completed, the water was drained from the flume with care to preserve the bed forms. Core samples were taken at each 4-foot station from station 10 to 50. The cores were then split into five 1-inch samples for analysis. Each sample was dried and weighed and the number of each color particle was determined by counting under ultraviolet light. The average weight of each particle was determined from previously determined curves of weight versus size for Ottawa sand. The concentration in ppm of each size could then be calculated from the relation

$$C = \frac{NW}{W_s} \tag{3-2}$$

where N is the number of particles in a sample, W is the weight per particle and W_s is the weight of the sample.

Chapter IV

ANALYSIS AND DISCUSSION OF RESULTS

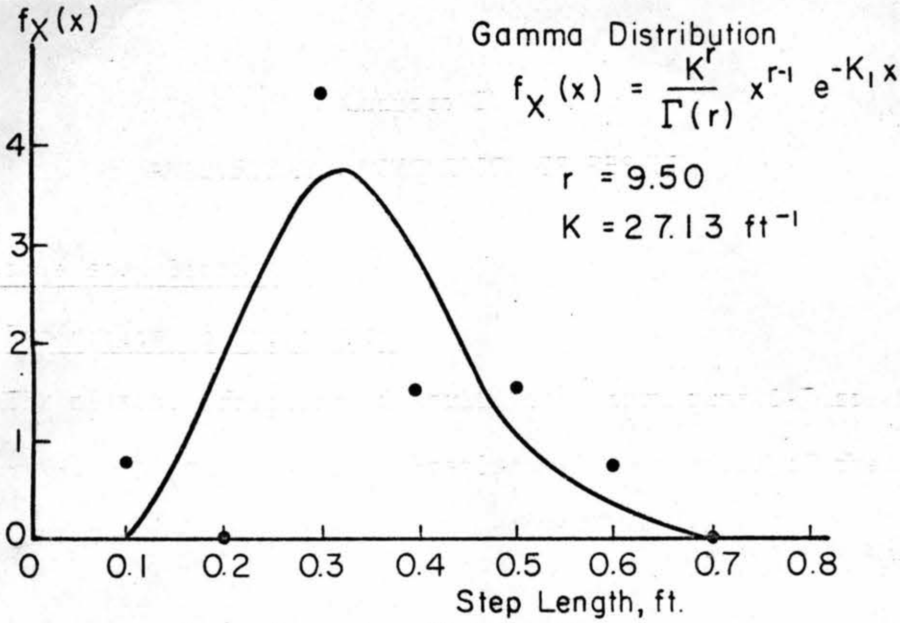
A. Particle step lengths

1. Probability distributions

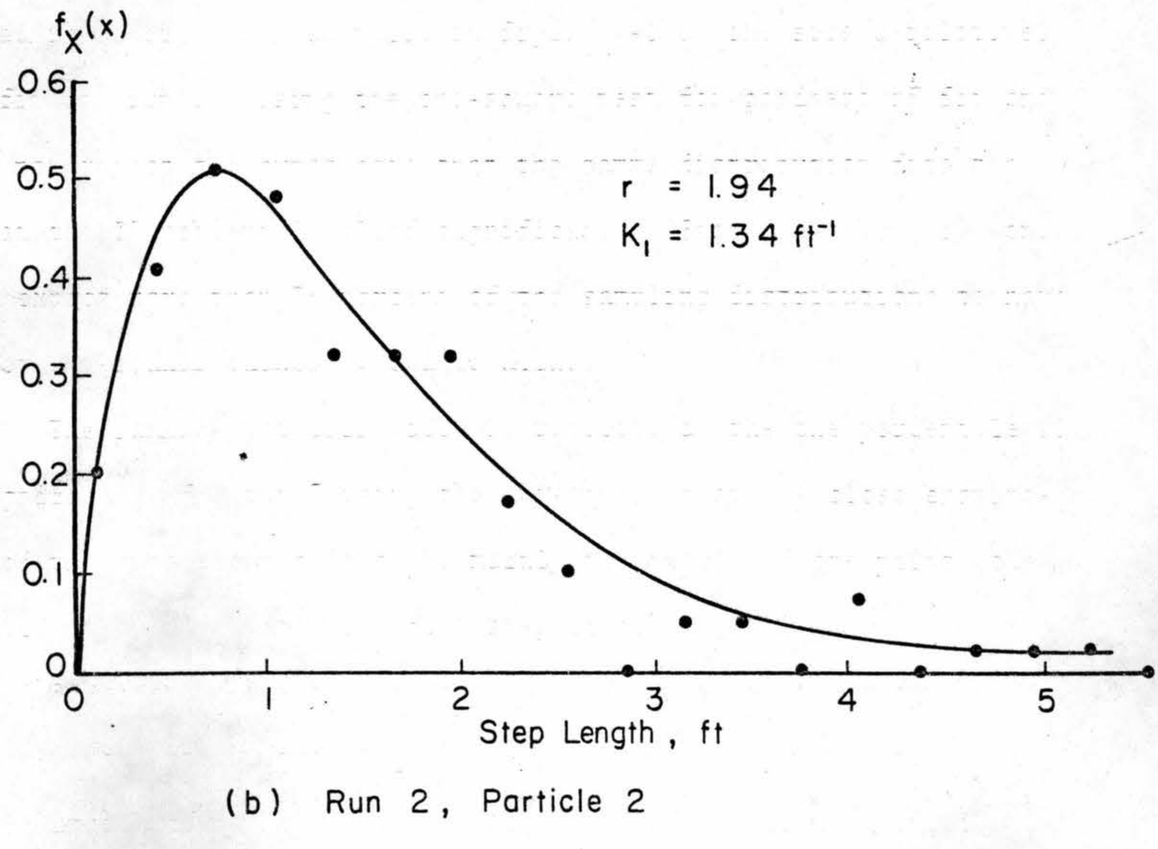
The measured frequency distributions from runs 1-7 are shown in figure 4-1, (a) through (g). Statistical properties of the data are given in table 4-1. Using the sample mean and variance, gamma distributions were fitted to the data using equations (2-28) and (2-29). The fit of the gamma distribution appears good in some cases and bad in others. For example, in figure 4-1(b) one sees a relatively good fit for run 2. Using the chi-square test for goodness of fit one would not reject the hypothesis that the gamma distribution fits the data at the 25 percent level of significance. Stated another way, one would expect more than 75 percent of the sampling distributions to have greater chi-square values by chance alone.

The gamma hypothesis would be rejected at the one percent level of significance for run 7 using the chi-square test. A close examination of the data shows this to be mainly the result of one point, the observed frequency at the 1.0 foot step length.

In general, the gamma distribution appears to represent adequately the step lengths. The bad fit, where it occurs, may be chiefly attributed to insufficient number of observations. A slightly better fit might be achieved by a distribution with three parameters so that the skewness could be considered. For the present, however, the two parameter gamma distribution appears quite adequate.

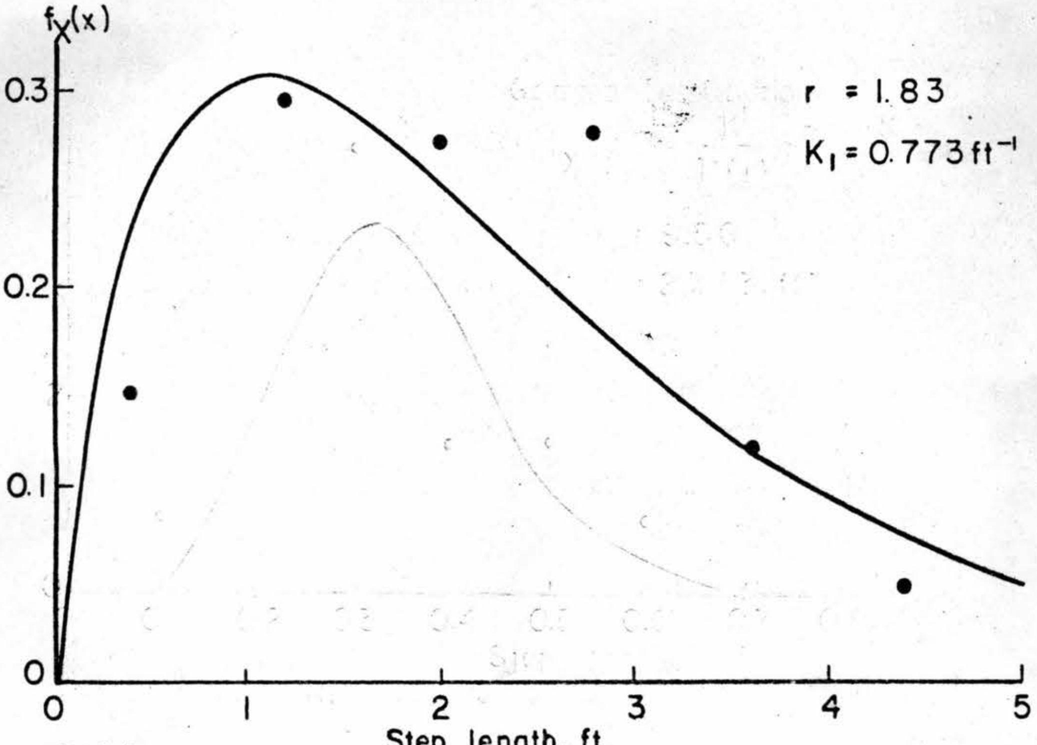


(a) Run 1, Particle 2

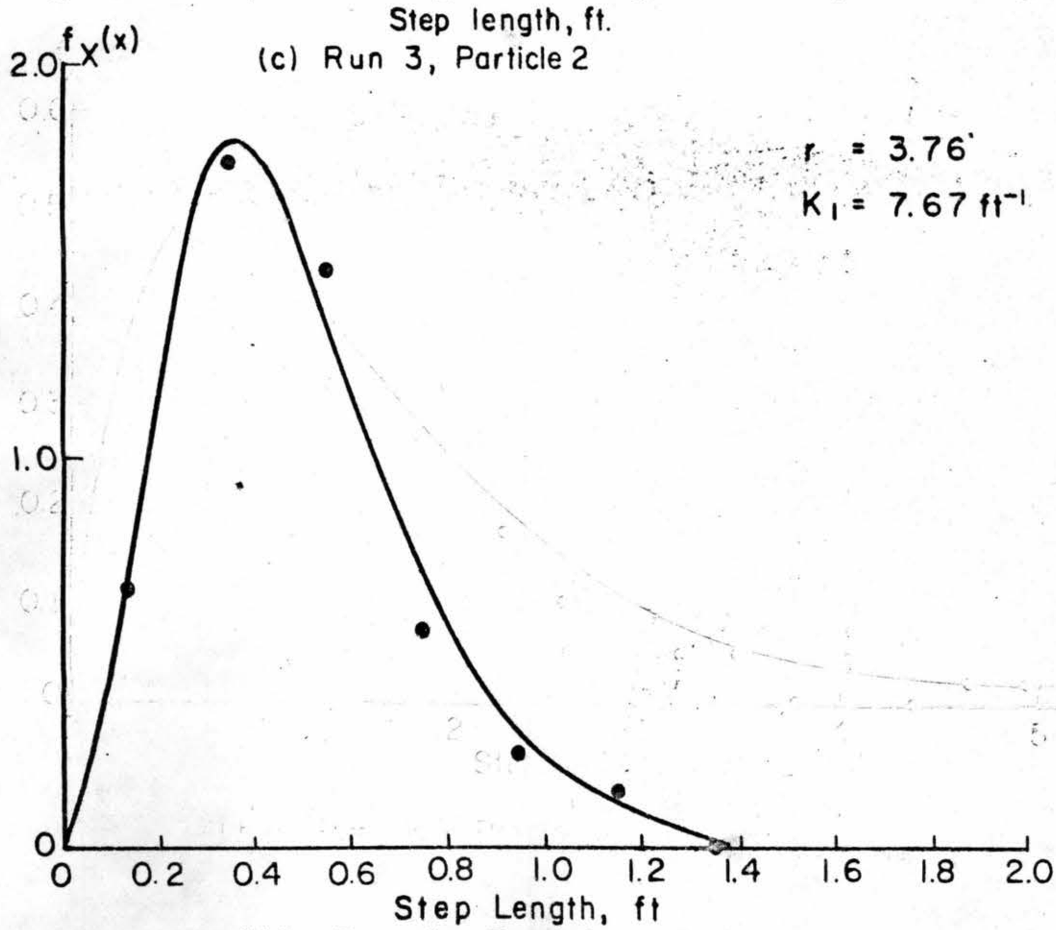


(b) Run 2, Particle 2

Figure 4-1. Step length probability distributions.



(c) Run 3, Particle 2



(d) Run 4, Particles 3, 6 and 7

Figure 4-1. Step length probability distributions.

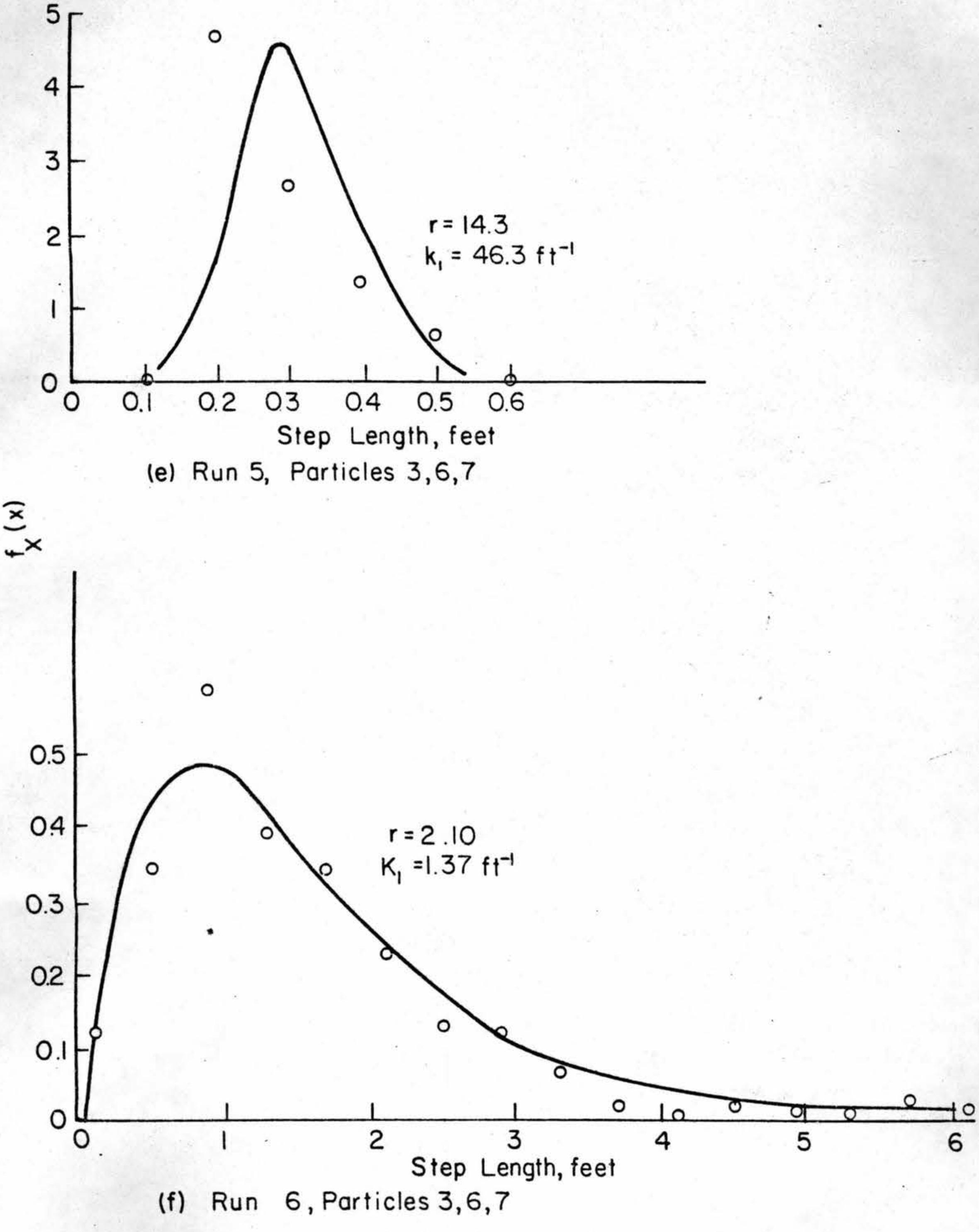
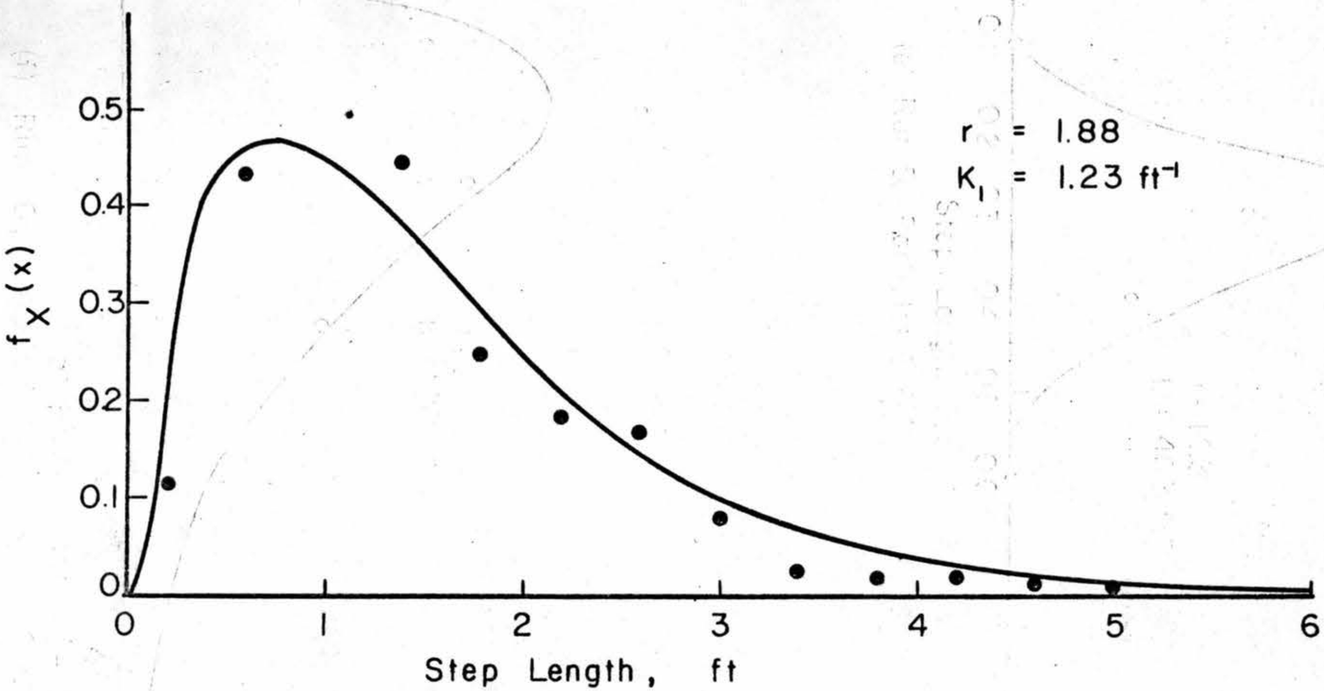


Figure 4-1. Step length probability distributions.



(g) Run 7, Particles 3, 6 and 7

Figure 4-1. Step length probability distributions.

TABLE 4-1. STATISTICAL PROPERTIES OF STEP LENGTHS AND REST PERIODS

Run No.	Particle No.	Step Length Data			Rest Period Data		
		Number of Data	Mean $\bar{X}$	Variance s_X^2	Number of Data	Mean $\bar{T}$	Variance s_T^2
1	2	12	0.35 ft	0.0129 ft^2	12	124.0 min	15727 min^2
2	2	117	1.45	1.08	113	21.6	1615
	7	56	1.33	.50	56	23.3	4484
3	2	176	2.39	3.14	158	11.9	312
	7	173	2.69	5.20	155	9.7	120
4	3	37	.52	.0769	33	26.8	495
	6	16	.47	.0929	13	32.1	1638
	7	50	.48	.0476	49	56.6	7297
	3+6+7	103	.49	.0639	95	42.9	4263
5	3	6	.31	------	6	91.9	------
	6	4	.30	------	4	125.0	------
	7	4	.31	------	4	81.3	------
	3+6+7	14	.31	.0067	14	98.3	10315
6	3	154	1.55	1.22	144	18.4	1050
	6	84	1.53	1.23	80	16.5	663
	7	94	1.49	.83	88	22.7	1234
	3+6+7	332	1.53	1.114	312	19.1	1002
7	3	95	1.35	.58	92	16.6	1507
	6	106	1.56	1.41	104	15.9	574
	7	79	1.71	1.76	69	15.4	209
	3+6+7	280	1.53	1.24	265	16.0	795

Some negative step lengths were noted. These were rare and small enough to be suspiciously near the limitations of the experimental equipment. Accordingly, the negative step lengths were not recorded as actual moves. They were absorbed into the next forward step.

2. Variation of step length with flow condition

The variation of mean step length with stream power for the seven runs is shown in figure 4-2. The relation appears to be exponential in the dune range. Using Shields' criterion one can show that initiation of motion should occur at a stream power value somewhere in the range between 0.001 and 0.003 ft-lb/sec-ft^2 for 0.33 mm particles and a flow depth of 0.5 feet. The variation in stream power is necessary because it is uncertain what the roughness should be. Using these values the relation in figure 4-2 is required to curve toward a zero step length at beginning of motion. In the dune range mean velocity, closely related to stream power, should affect considerably the step length once a particle was entrained. In the lower ripple range, the criteria of bed shear is more important. According to the data shown in figure 4-2 there is not a significant difference between the relations for the 0.45 mm and the 0.33 mm particles. One might explain this in that these sizes are rather close, perhaps too close to differentiate. This does support the conclusion that step length is less related to particle size than to dune length.

Figure 4-3 shows the relation of variance of step length to stream power. The lower range appears well represented by a power function whereas the curve begins to top out at higher shears. It is expected that the relation between stream power and step length

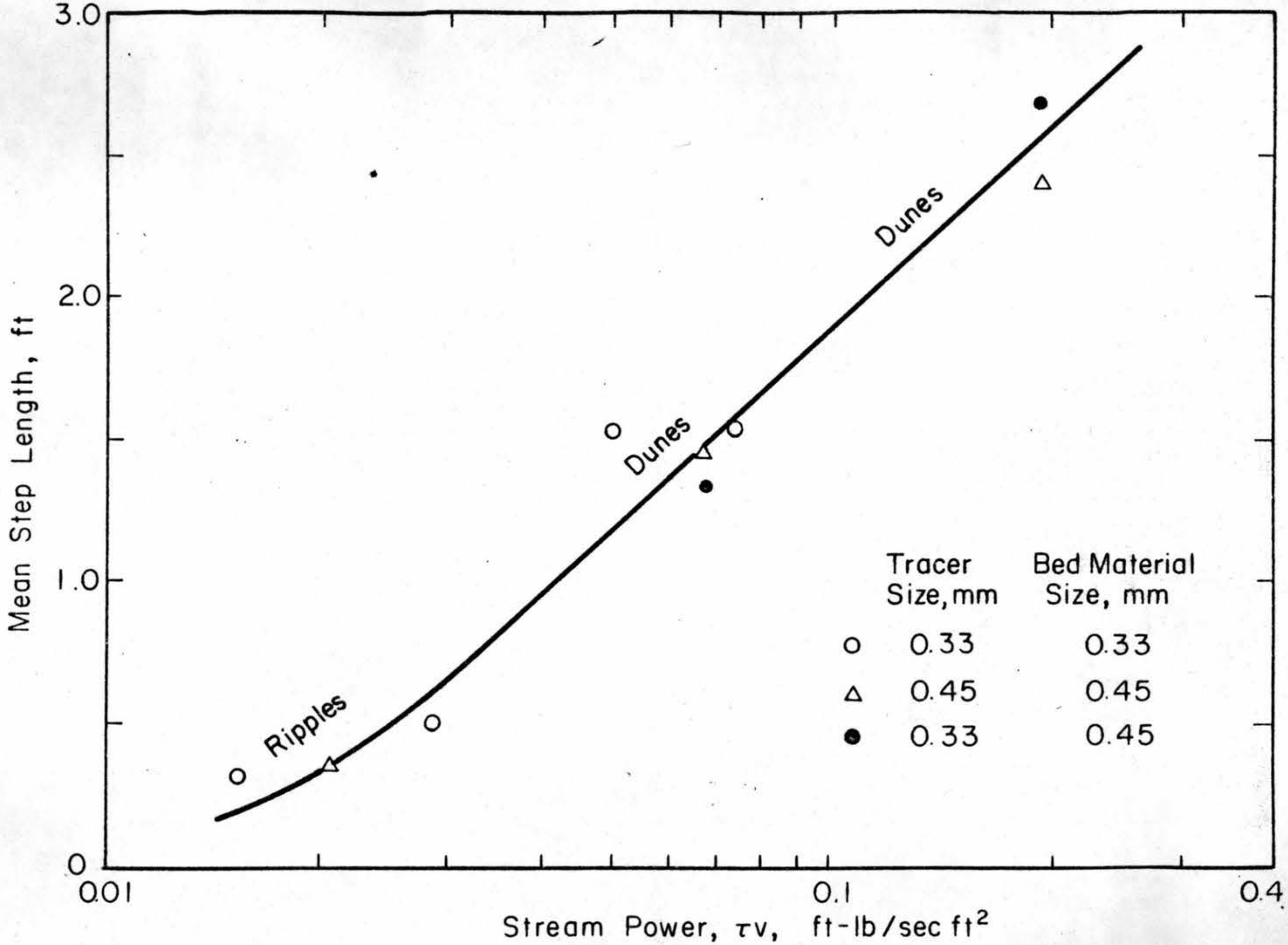


Figure 4-2. Relation between mean step length and stream power.

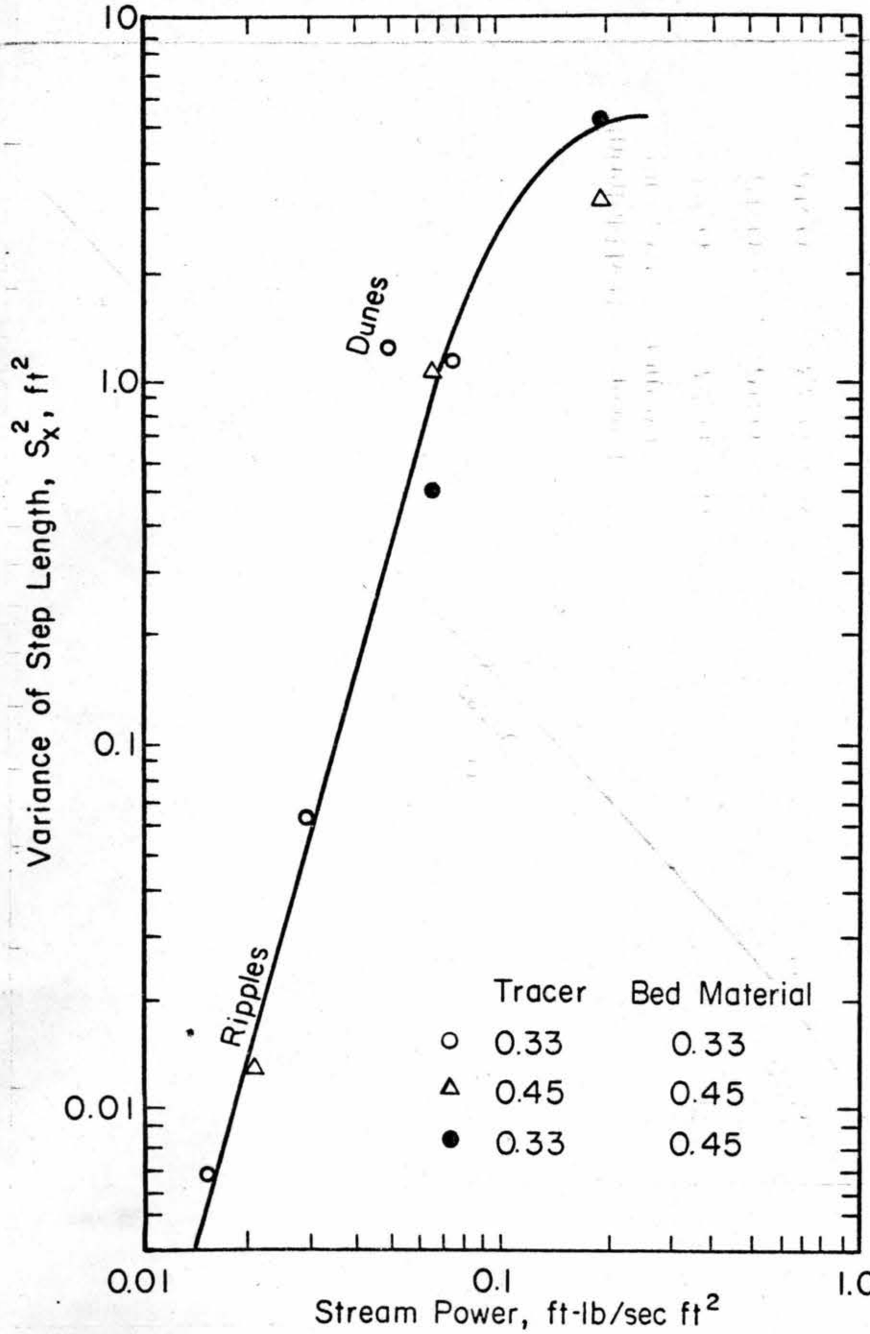


Figure 4-3. Relation between the variance of step length and stream power.

variance should change near the point where the transition range is neared since the mechanism of particle movement changes. When the bed form goes to plane bed, the step lengths should increase greatly.

From the relations given in figures 4-2 and 4-3, one can determine the parameters of the gamma distributions governing step lengths for all flow ranges within the range of the figures, from ripples through dunes. The relations are applicable to the two foot flume, the particular bed material used and the flow depths near 0.5 feet. If these relations could be generalized and if stream power were the correct correlating parameter, one would be able to calculate step length distributions directly from a knowledge of flow conditions and bed material.

3. Effect of particle size and gradation of bed material

The difference between 0.33 mm and 0.45 mm is too small to define clearly what effect size of bed material or tracer has on the distribution of step lengths. In the ranges tested there is no apparent difference. There appears to be little difference between the mean step lengths for 0.33 or 0.45 mm particles, whether the bed material is of one size or the other. It should be noted that for the 0.45 mm bed material the 0.33 mm tracers correspond to the d_{22} size, or the size for which 22 percent of the bed material by weight is finer whereas the 0.45 mm is the d_{50} size. Relatively speaking then, there is a significant difference between the sizes, but with such uniform sands it is not likely that the difference will affect the behavior of the tracers.

There should be an effect of the bed material size and gradation on the step lengths. The step lengths must be related to bed form characteristics which are, in turn, related to regime of flow and bed material properties (Simons and Richardson, 1966).

Goswami (1967) has shown that, for a constant mean velocity of flow, ripple length increases and dune length decreases with size of bed material, other factors being constant. The data upon which Goswami based his conclusions were limited, however, and it is unknown whether his conclusions may be generalized. One can say, however, that bed material size and gradation are parameters which affect bed form characteristics.

B. Rest periods

1. Unconditional rest periods

The rest period distributions may be measured directly, or estimated from bed form data. Figure 4-4, (a) through (g), shows the directly measured rest period distributions with exponential distributions fitted by the relation

$$k_2 = \frac{1}{\bar{T}} \quad . \tag{4-1}$$

The fit of the exponential distribution is not extremely good. In general, the chi-square test shows the data deviating from the theoretical distribution more than it should at the 5 percent level of significance. The exponential distribution does a better job, however, in fitting the data than does the gamma distribution. It is believed that a significant portion of the deviations may be explained by the limited number of observations and by the limitations of the

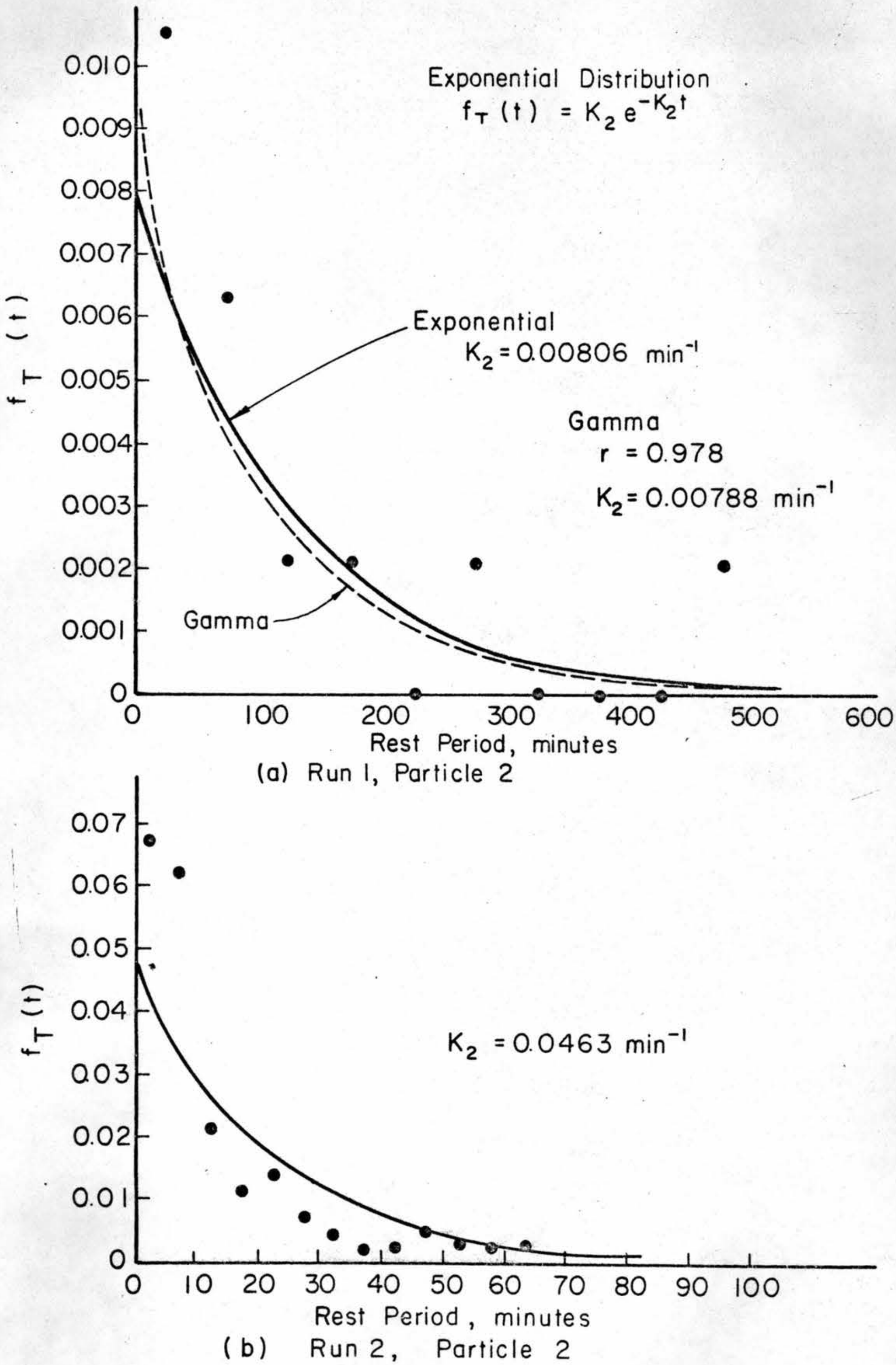


Figure 4-4. Measured distributions of rest periods.

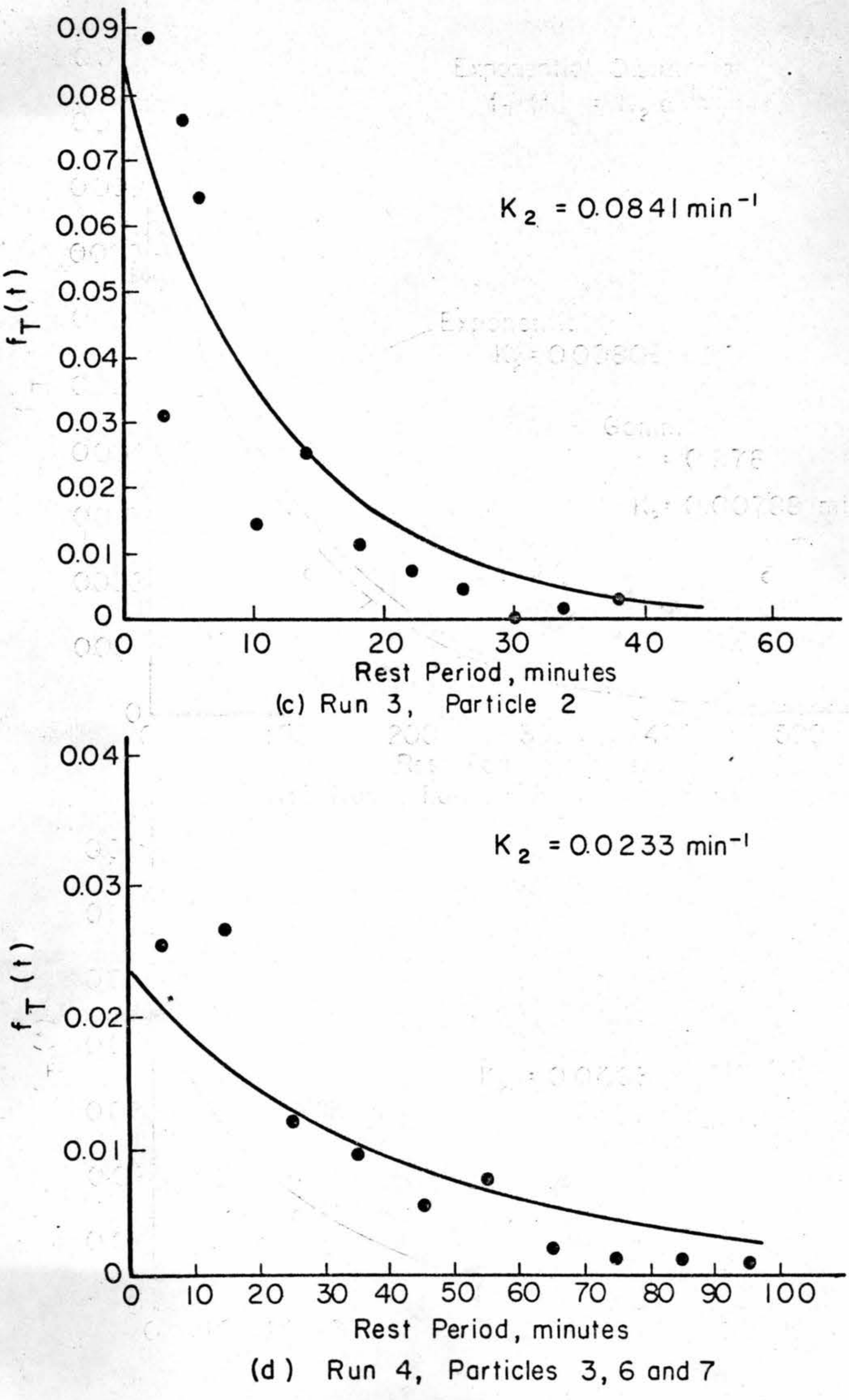
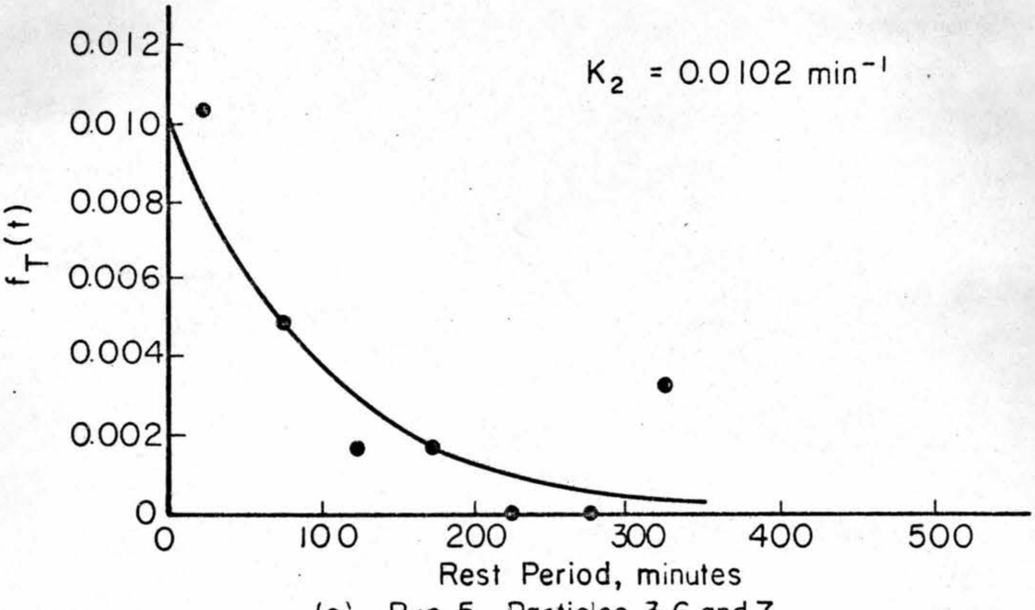
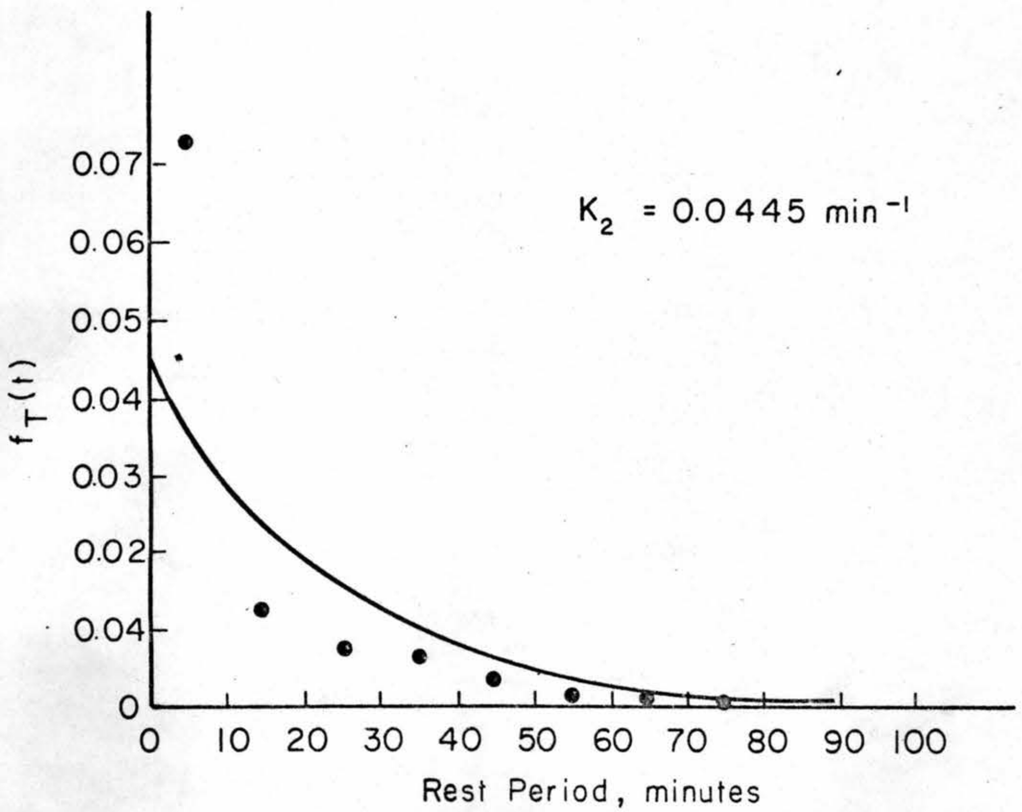


Figure 4-4. Measured distributions of rest periods.

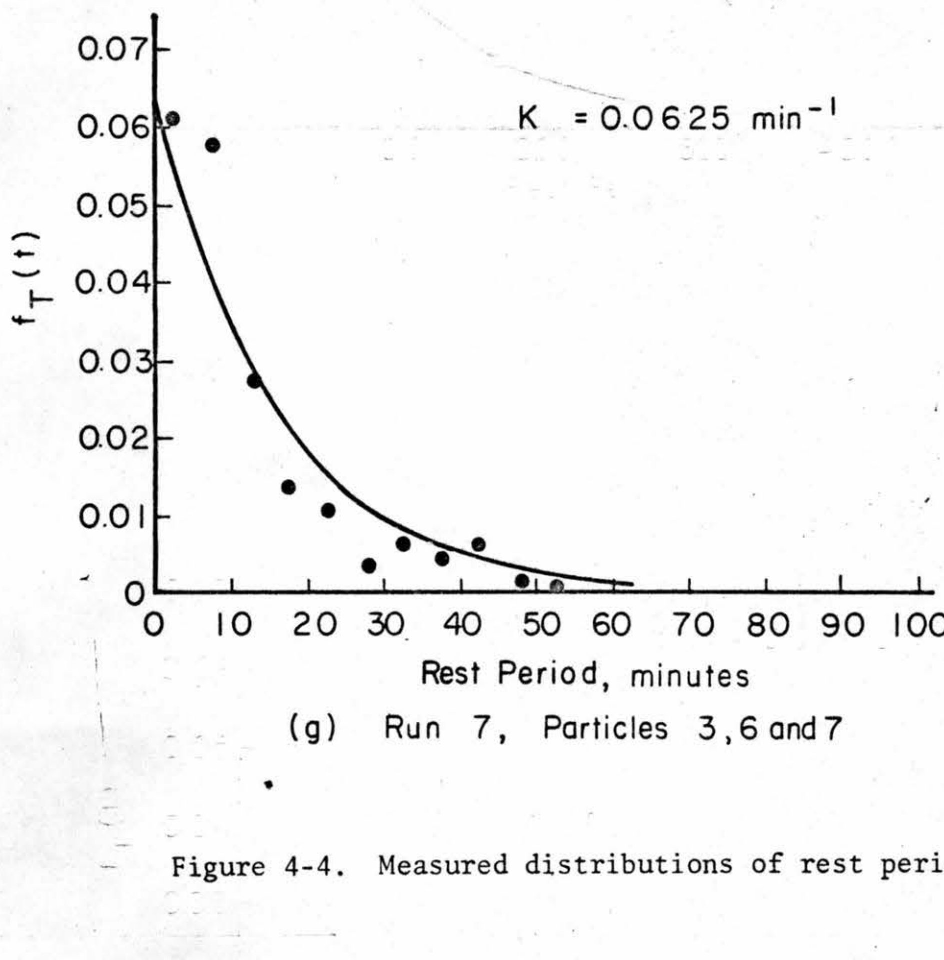


(e) Run 5, Particles 3,6 and 7



(f) Run 6, Particles 3, 6 and 7

Figure 4-4. Measured distributions of rest periods.



(g) Run 7, Particles 3,6 and 7

Figure 4-4. Measured distributions of rest periods.

experimental equipment. The remaining deviations might require a distribution with more degrees of freedom than the gamma.

2. Conditional rest periods

Conditional rest period distributions were established from frequency analyses of the temporal bed form data. The length of an upward excursion of the process $y(t)$ was taken as the duration of a rest period, given that the particle was deposited at a fixed level $H(t)$.

Figure 4-5 (a) and (b) shows the frequency distributions for conditional rest periods for runs 4 and 6. The exponential distribution fits the data well. The gamma distribution could as well be used but the shape parameter r would be close to one indicating that the simpler exponential function should be used.

The mean and variance of the conditional distributions $f_{T|Y}(t|y)$ relate to bed level $H(t)$, measured in terms of standard deviations as shown in figures 4-6 and 4-7. For the range from -2.0 to +2.0 standard deviations about mean bed elevation both the mean and variance relate to bed level as linear functions on semi-log paper, or in other words, the relations

$$\bar{T}|Y = \alpha_1 \, e^{-\beta_1 \, y} \tag{4-2}$$

and

$$s_T^2 = \alpha_2 \, e^{-\beta_2 \, y} \tag{4-3}$$

hold. The runs not shown in figures 4-5, 4-6 and 4-7 demonstrated bed properties similar to those of runs 4 and 6. There should be deviations from the exponential relations evident in figures 4-6 and 4-7 above +2.0 standard deviations and below -2.0 standard deviations from mean

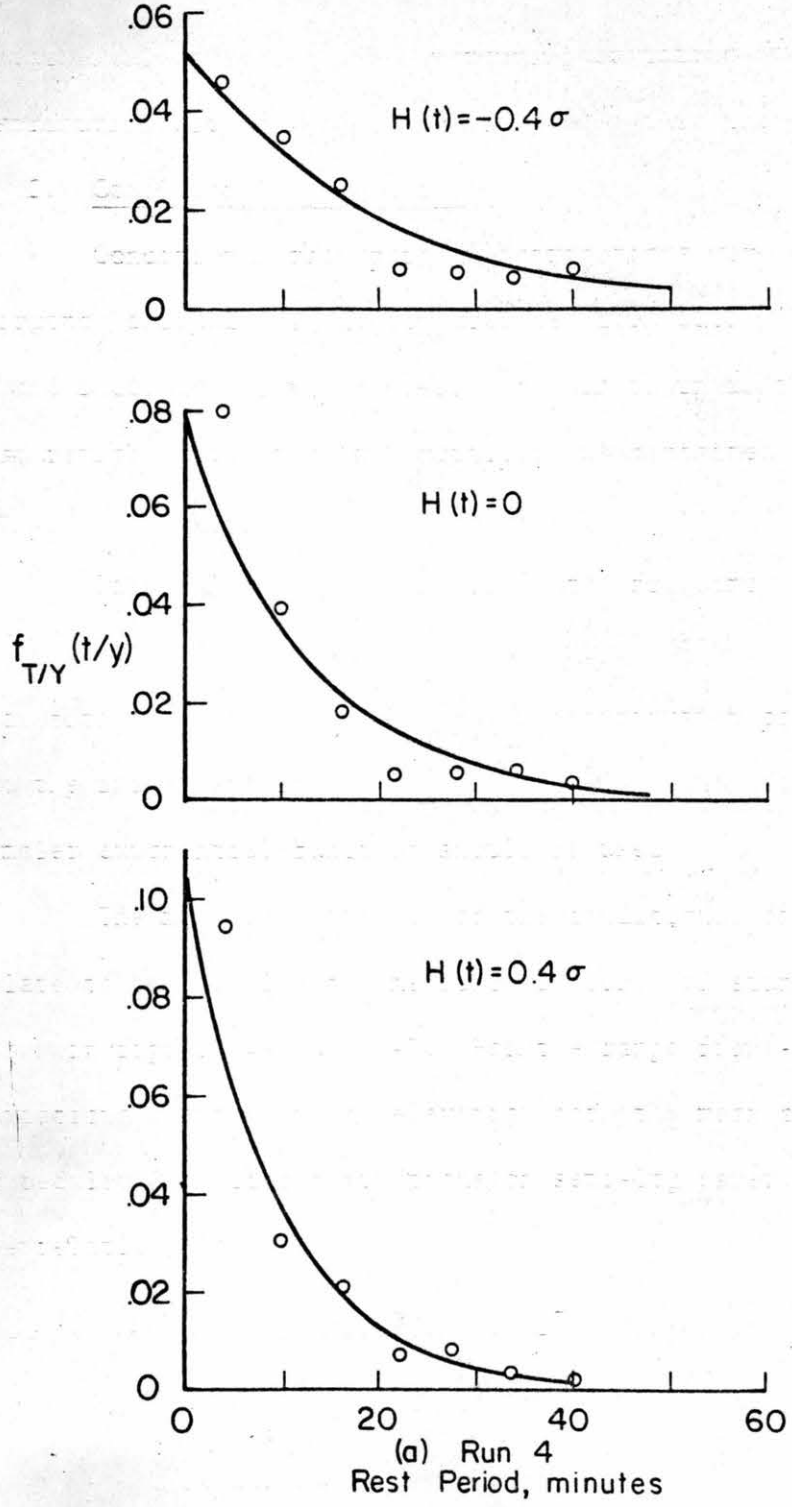


Figure 4-5(a). Conditional rest period distributions from bed form data for Run 4.

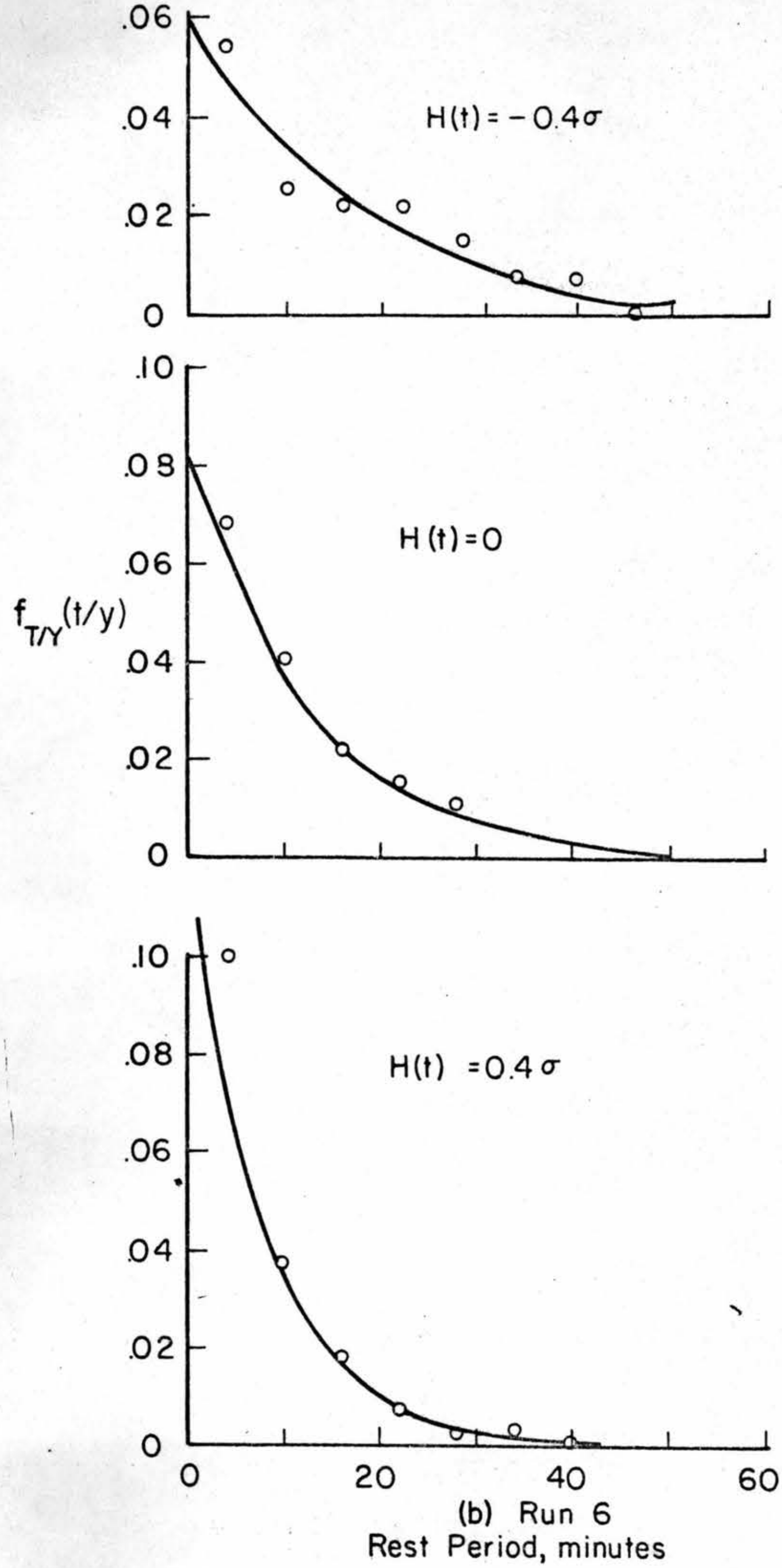


Figure 4-5(b). Conditional rest period distributions from bed form data for Run 6.

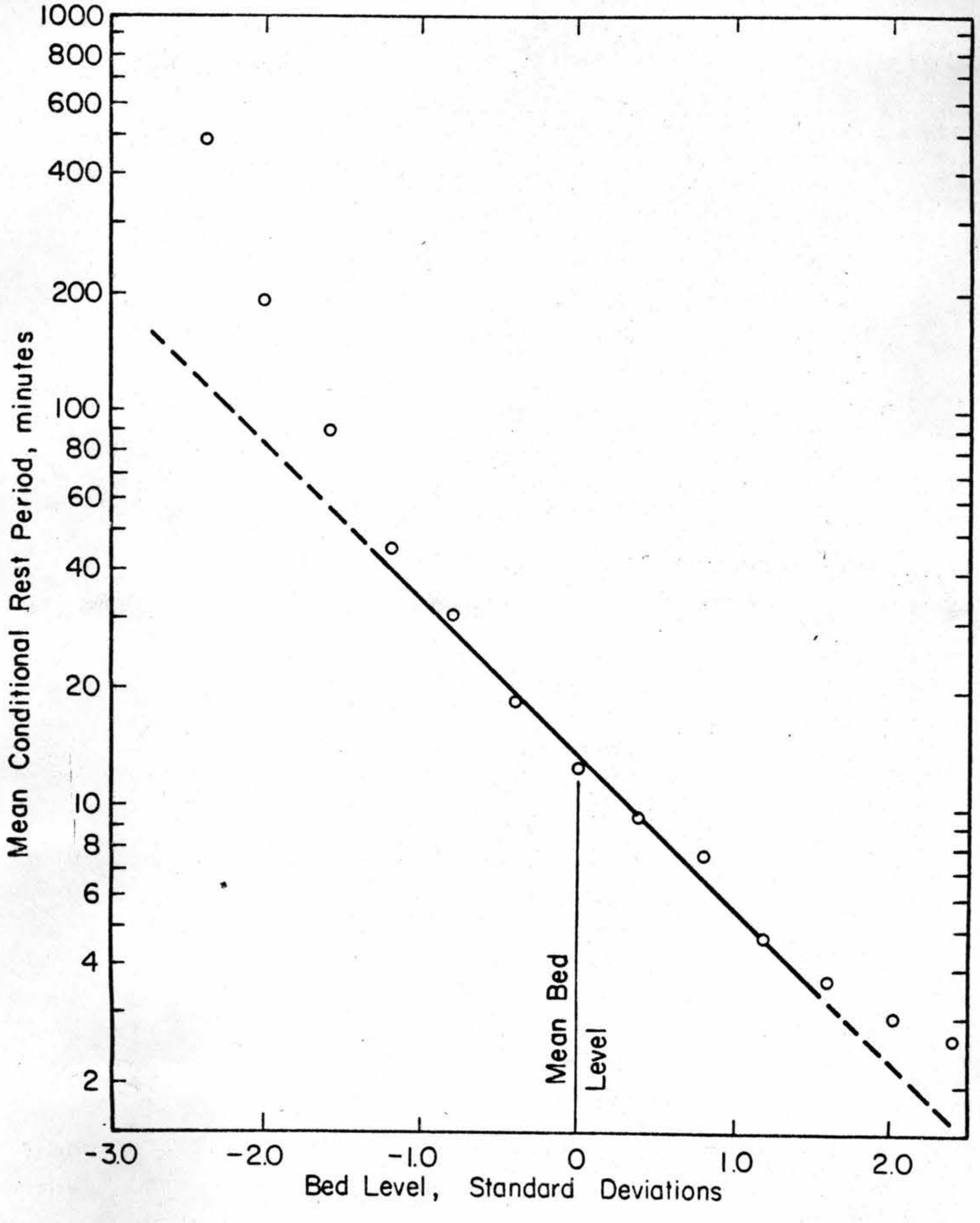


Figure 4-6(a). Relation between mean conditional rest period and bed elevation for Run 4.

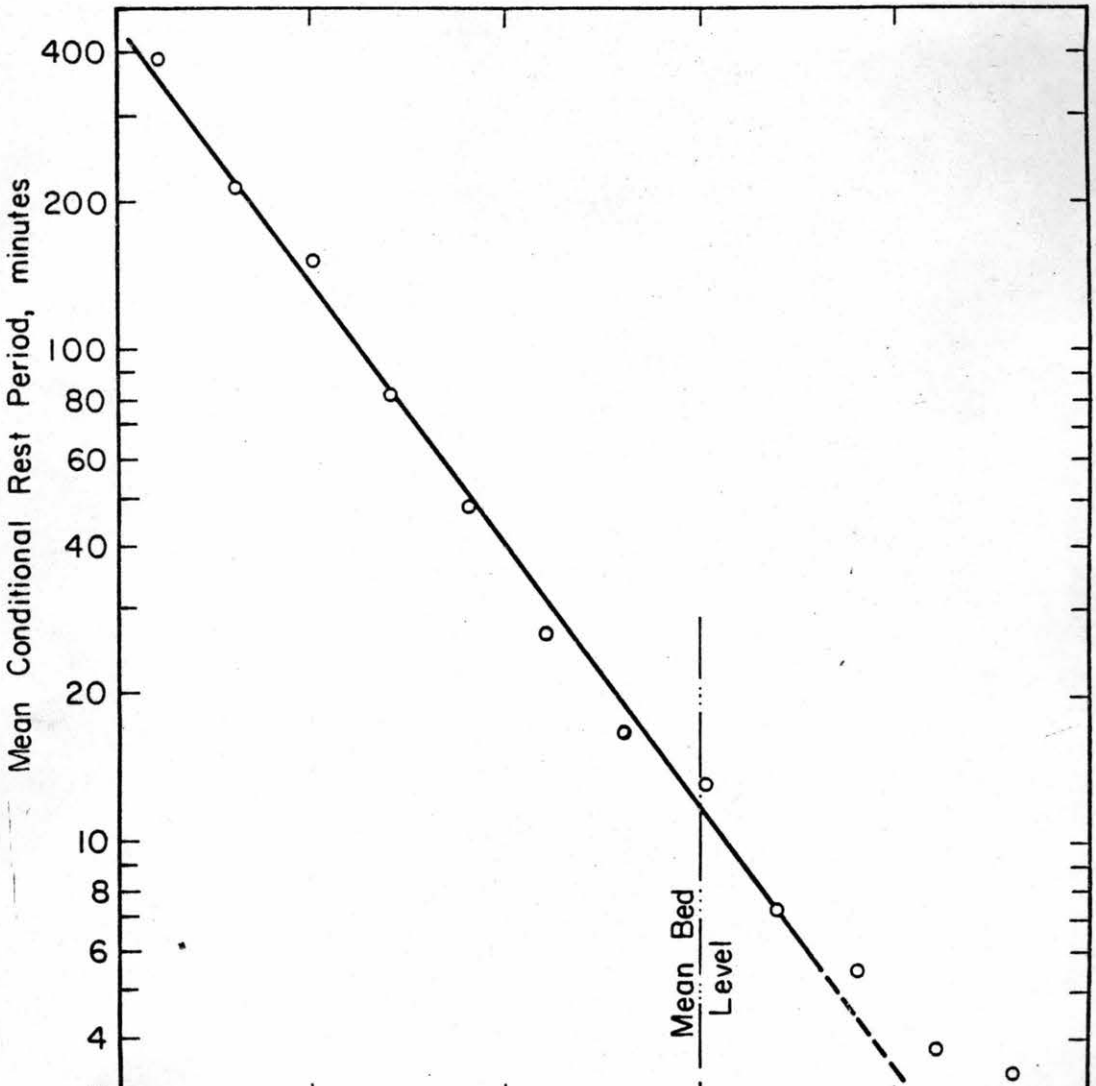


Figure 4-6(b). Relation between mean conditional rest period and bed elevation for Run 6.

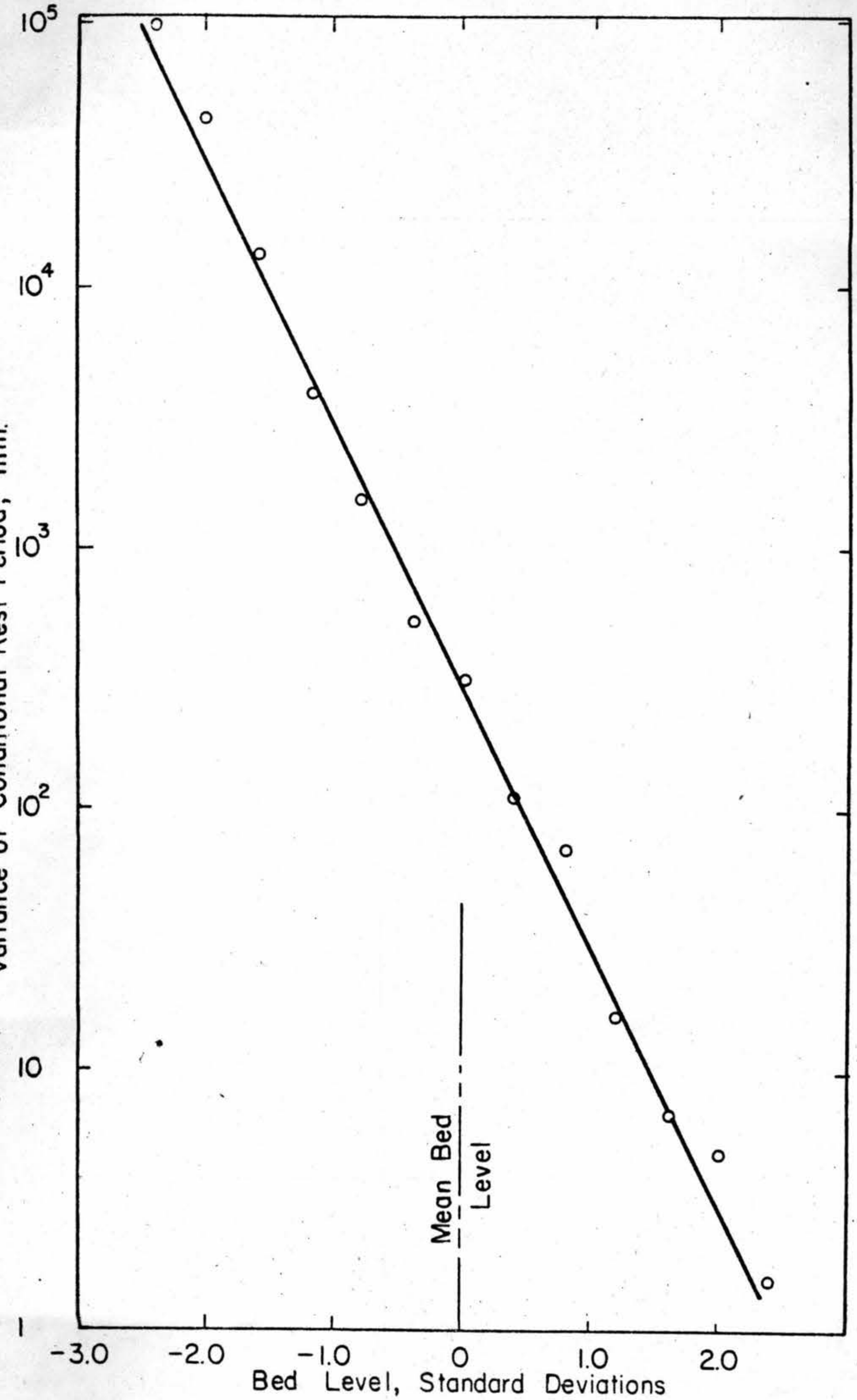


Figure 4-7(a). Relation between variance of conditional rest period and bed elevation for Run 4.

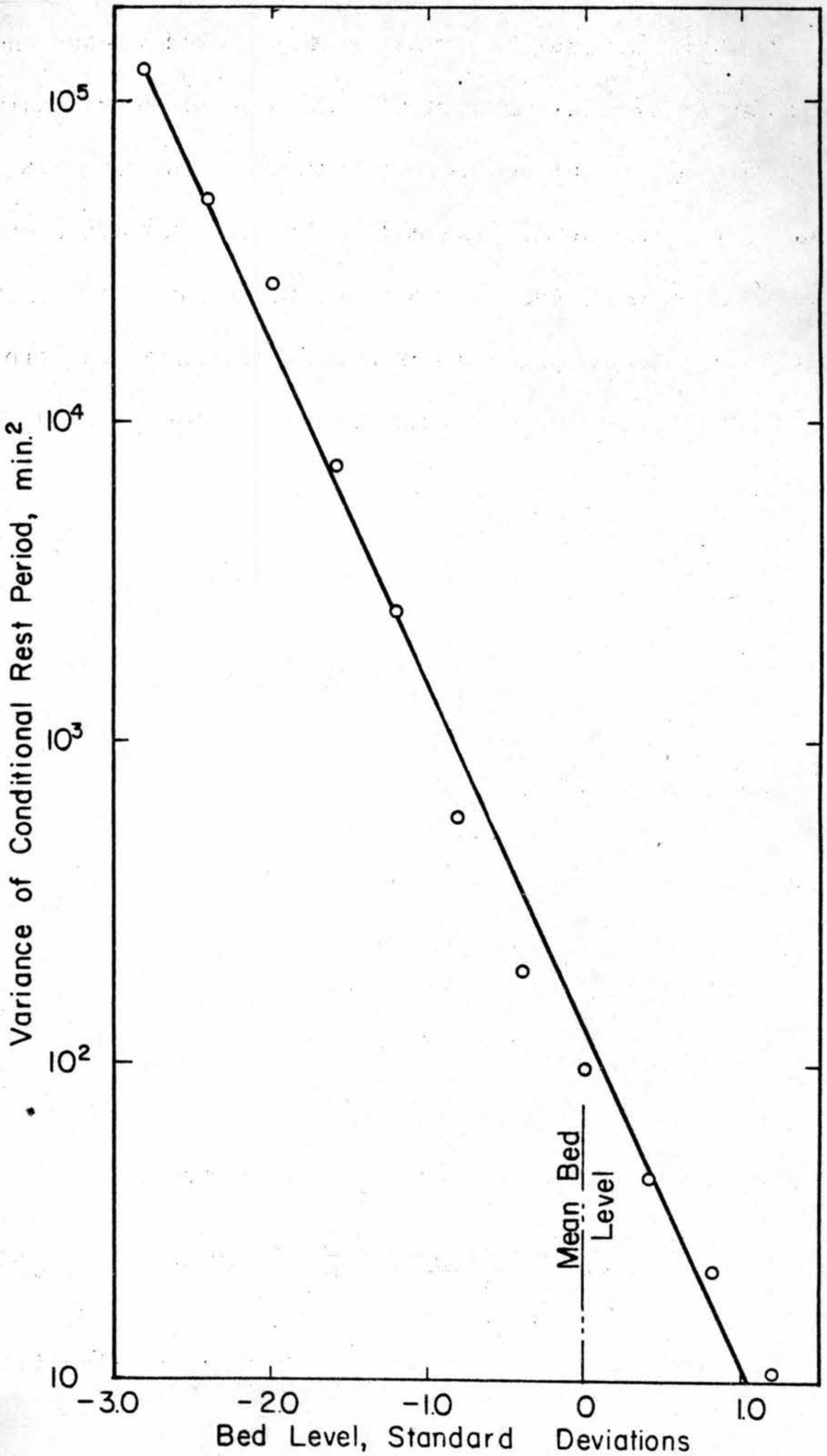


Figure 4-7(b). Relation between variance of conditional rest period and bed elevation for Run 6.

bed elevation. The height and depth of bed form crests and troughs are subject to finite physical limitations such that the conditional rest periods for high values of bed level should approach zero and those for low values should approach infinite duration. Insufficient length of record was observed to establish these trends in the present experiments.

Using the relations for mean and variance conditional distributions for $f_{T|Y}(t|y)$ can be established. For the gamma distribution,

$$r = \frac{\bar{T}^2}{s_T^2} = \frac{{\alpha_1}^2}{\alpha_2} e^{-(2\beta_1-\beta_2)y} \tag{4-4}$$

and

$$k_2 = \frac{\bar{T}}{s_T^2} = \frac{\alpha_1}{\alpha_2} e^{-(\beta_1-\beta_2)y} . \tag{4-5}$$

For simplification, let

$$A = \frac{\alpha_1}{\alpha_2} \tag{4-6}$$

$$B = \beta_1-\beta_2 \tag{4-7}$$

$$C = \frac{{\alpha_1}^2}{\alpha_2} \tag{4-8}$$

$$D = 2\beta_1-\beta_2, \tag{4-9}$$

then the gamma distribution for $f_{T|Y}(t|y)$ becomes

$$f_{T|Y}(t|y) = \frac{A\ C\ t \exp - (Dy+1)}{\Gamma\left(C e^{-Dy}\right)} \exp - \left(A e^{-By} + BC\ y\ e^{-Dy}\right) . \tag{4-10}$$

If instead, r is taken to be one, the distribution becomes

$$f_{T|Y}(t|y) = k_2(y)\ e^{-k_2(y)\ t} \tag{4-11}$$

where

$$k_2(y) = \frac{1}{E(T|Y)} = \frac{1}{\alpha_1} e^{\beta_1 y} . \tag{4-12}$$

This value for k_2 is quite similar to that given by Mielke (personal communication) and presented as $b(y)$ in equation (2-41)

$$f_{T|Y}(t|y) = b(y)\ e^{-t\ b(y)} \tag{2-41}$$

where

$$b(y) = 2^{y/2\sigma} . \tag{2-46}$$

Note that the value assumed by Mielke for $b(y)$ and that determined by measurement for $k_2(y)$ differ only by a constant multiplier. It was shown that Mielke's model led to a quasi-exponential distribution for $f_T(t)$ so that the use of equation (4-10) in equation (2-11) could be expected to yield similar results.

C. Relations between flow conditions and bed properties

Often sediment studies report bed form characteristics such as the length and height of the bed features. Prior to the advent of the sonic sounder and automatic data processing equipment the characteristics were usually measured by eye with the mean value of several observations being reported. In this study mean values are based on many observations and computer analysis of the data. There is available some similar data taken under different conditions. This data, along with data taken in the present study, is shown in figure 4-8 which is the relation between mean dune length and stream power. The relation appears to be quite definitive with the range of flow conditions varying from those in the 2-foot flume through field conditions.

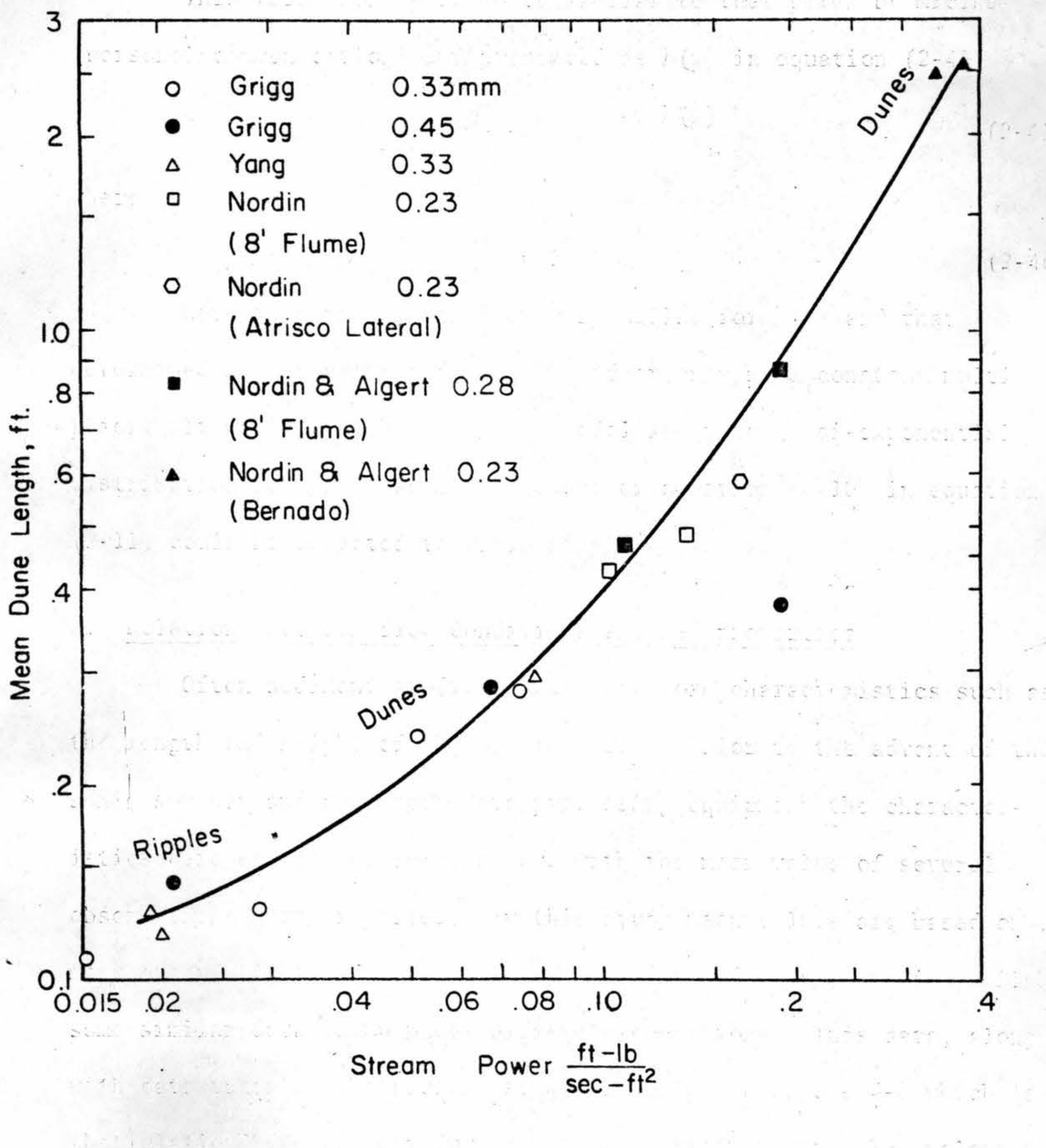


Figure 4-8. Relation between mean bed form length and stream power.

In figure 4-9 is shown the relation between standard deviation of bed form height and stream power. The standard deviation is used because it is an easy quantity to measure and is a characteristic measure of a bed profile. Assuming a Gaussian distribution of bed elevation, the average height of a bed form (trough to crest) will be approximately 1.33 standard deviations. Most of the data shown in figure 4-9 is for rather fine sands with fixed gradations. Some indication of how the relation might proceed is inferred by the extension of the relations for the 0.45 mm sand and the 0.28 mm sand. Although there is not sufficient data to define these relations, such inferences agree with the observations of Simons and Richardson (1966) and Goswami (1967) that bed material size affects the size and formation of dunes. It is interesting to note on figure 4-9 that the point of maximum curvature in the relation occurs at approximately a stream power of 0.07 ft-lb/sec-ft^2. This value agrees with the break point on Simons and Richardson's (1966) bed form prediction chart for bed materials in the 0.2-0.3 mm range. The results shown in figure 4-9 are a limited confirmation of Simons and Richardson's bed form chart.

It is noteworthy that the data shown on figures 4-8 and 4-9 exhibit less scatter than most sediment data. The lack of scatter is attributed to the absence of human judgment in measuring the lengths and heights of the bed forms and to the large number of observations made.

Both the distribution of step lengths and dune lengths follow approximately the gamma distribution. Figure 4-10, (a) through (g), shows the experimental data of dune length frequency distributions

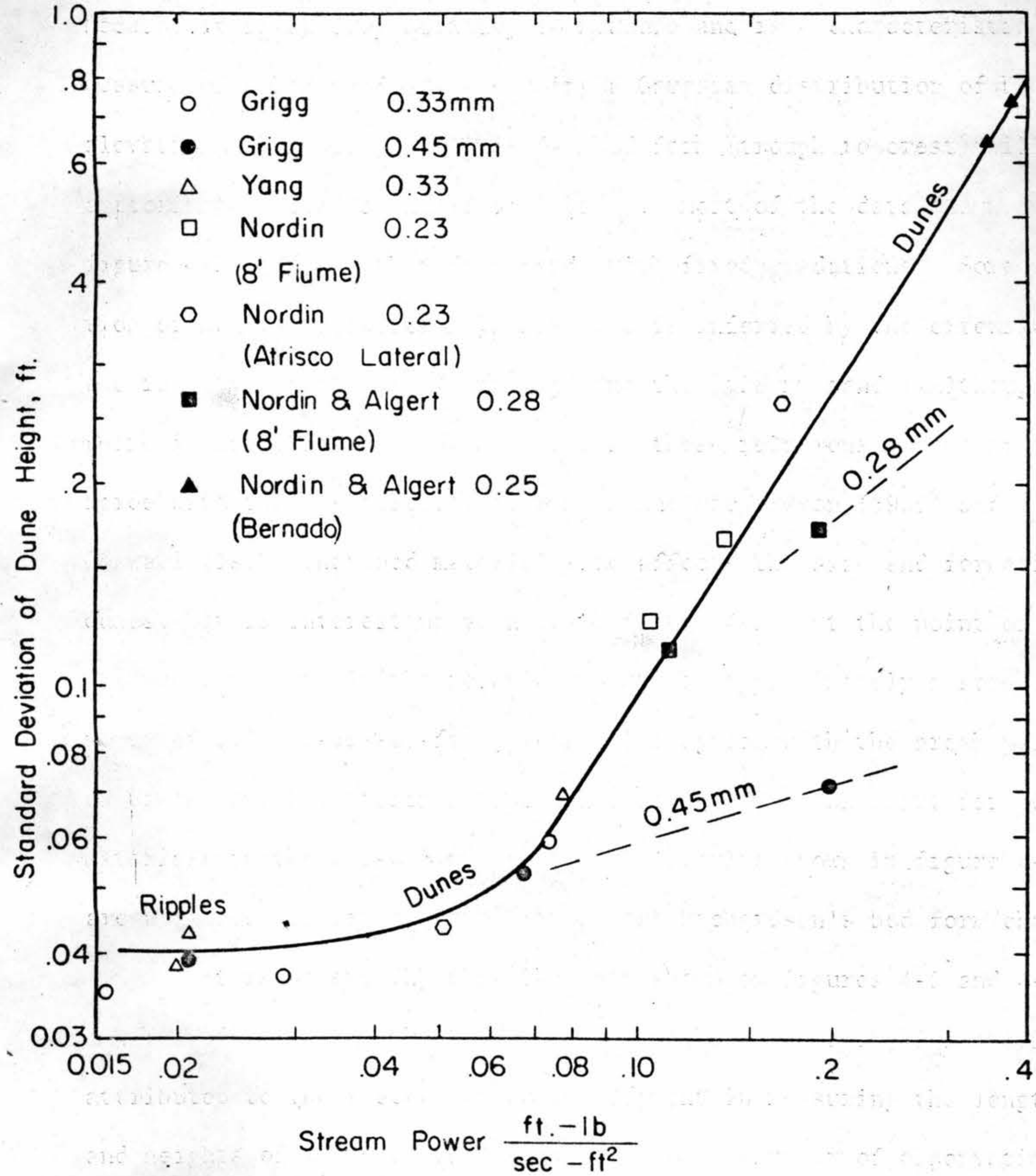
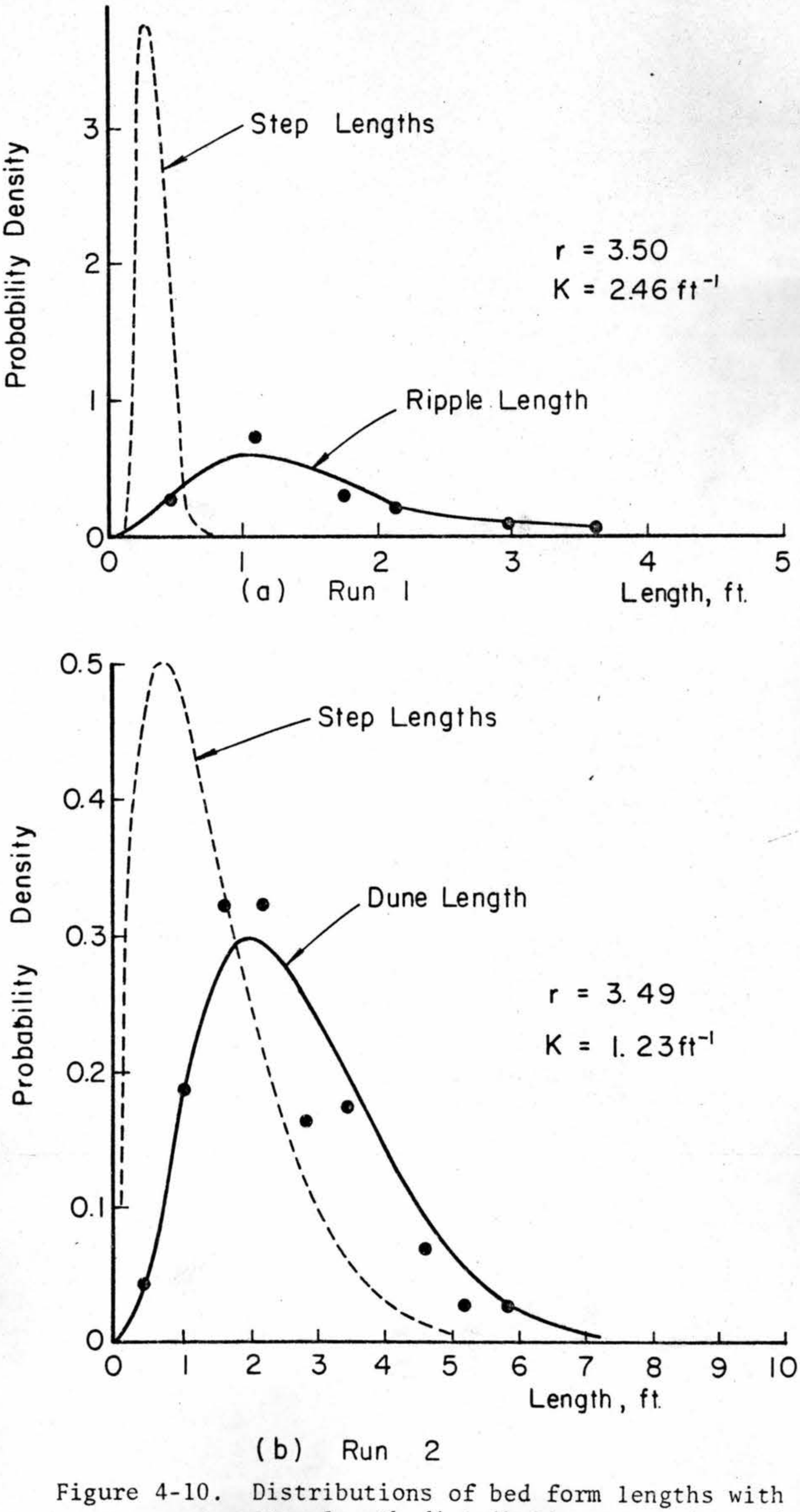


Figure 4-9. Relation between standard deviation of bed form height and stream power.



(b) Run 2

Figure 4-10. Distributions of bed form lengths with step length distributions superimposed.

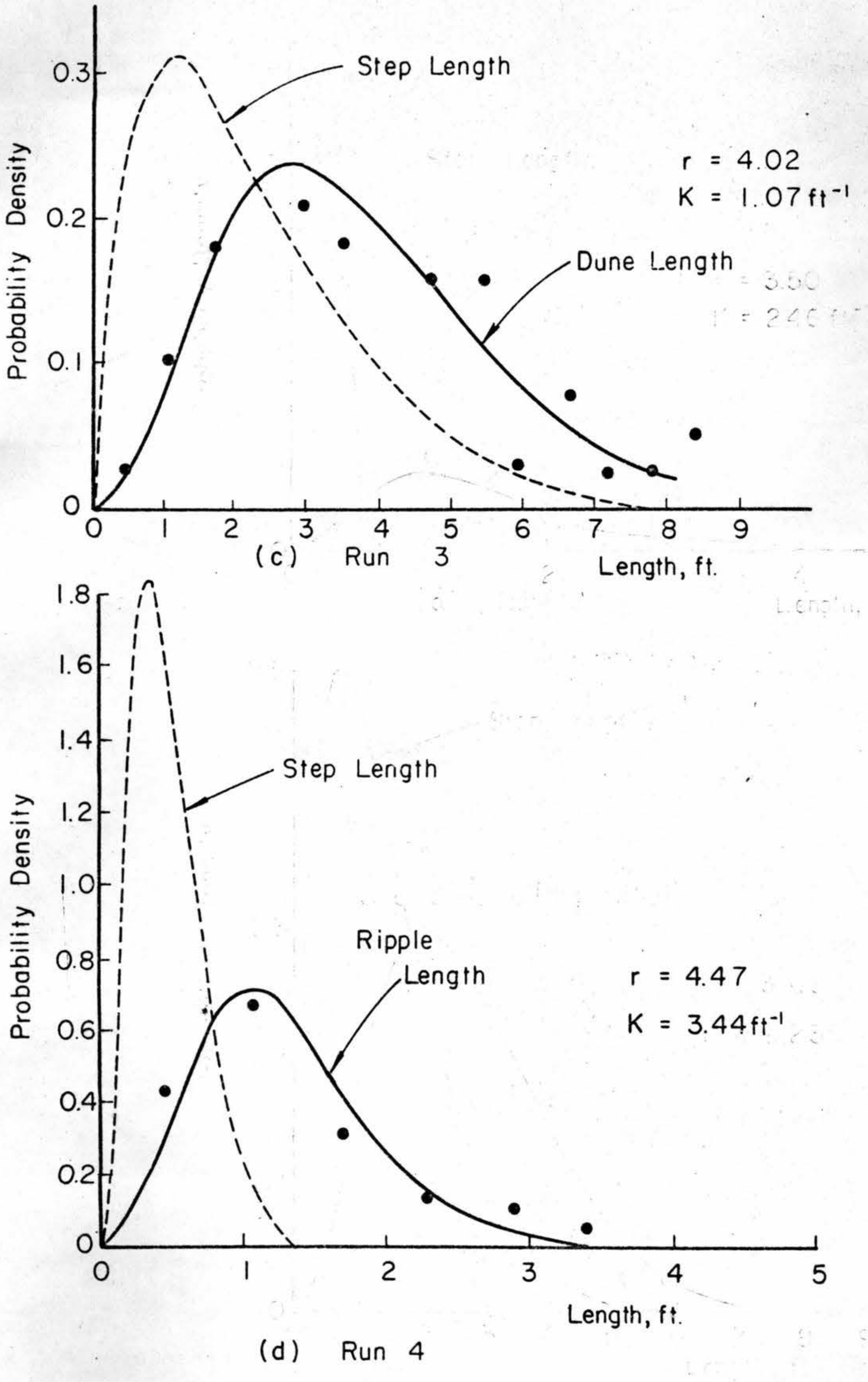
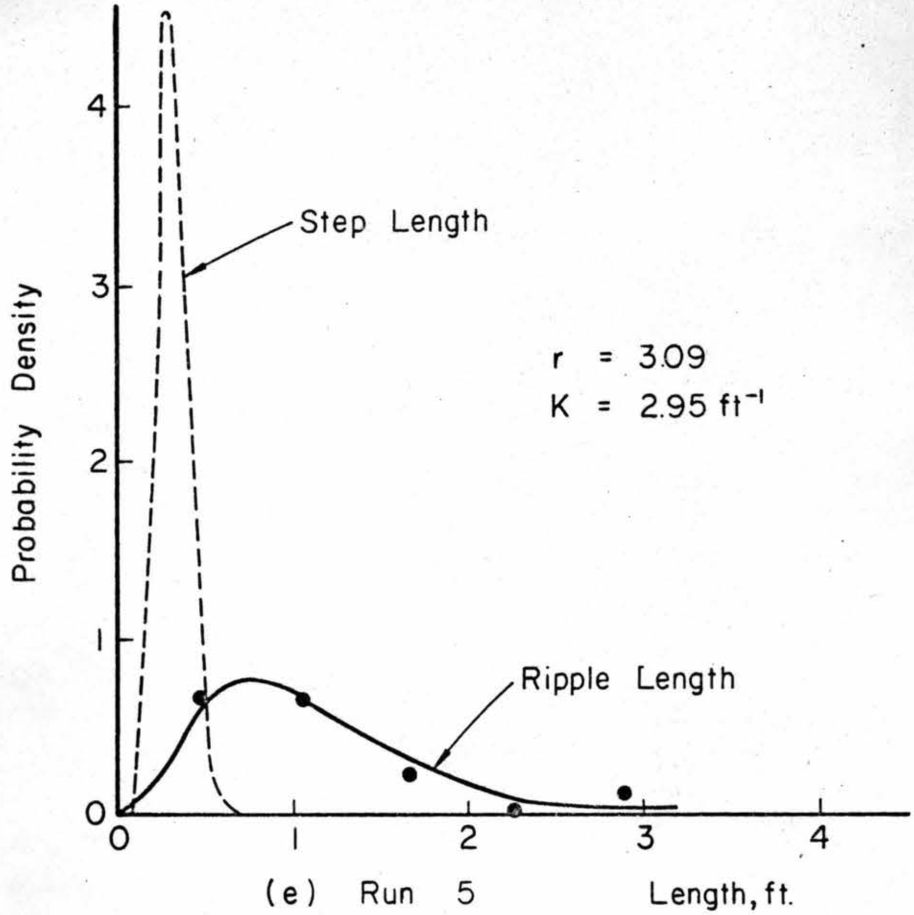


Figure 4-10. Distributions of bed form lengths with step length distributions superimposed.



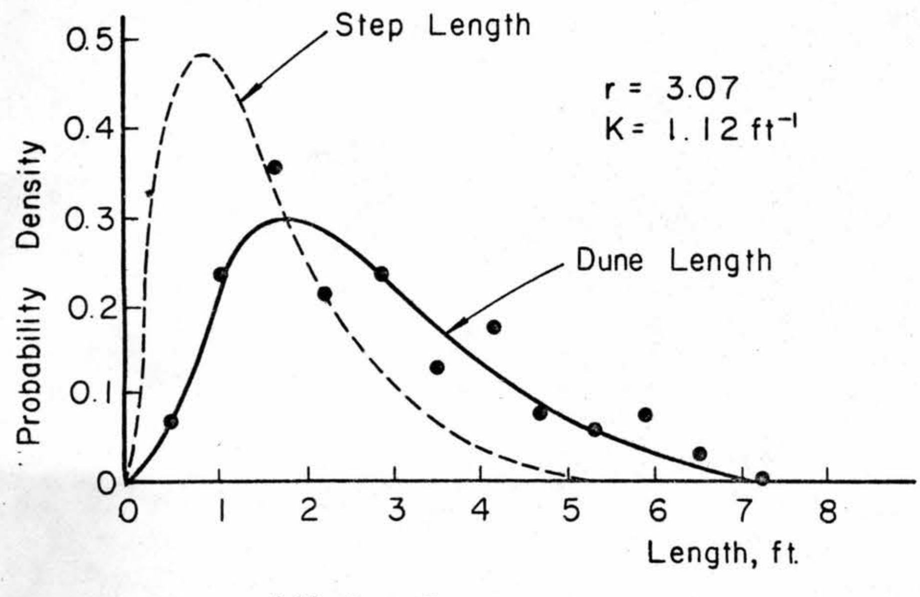


Figure 4-10. Distributions of bed form lengths with step length distributions superimposed.

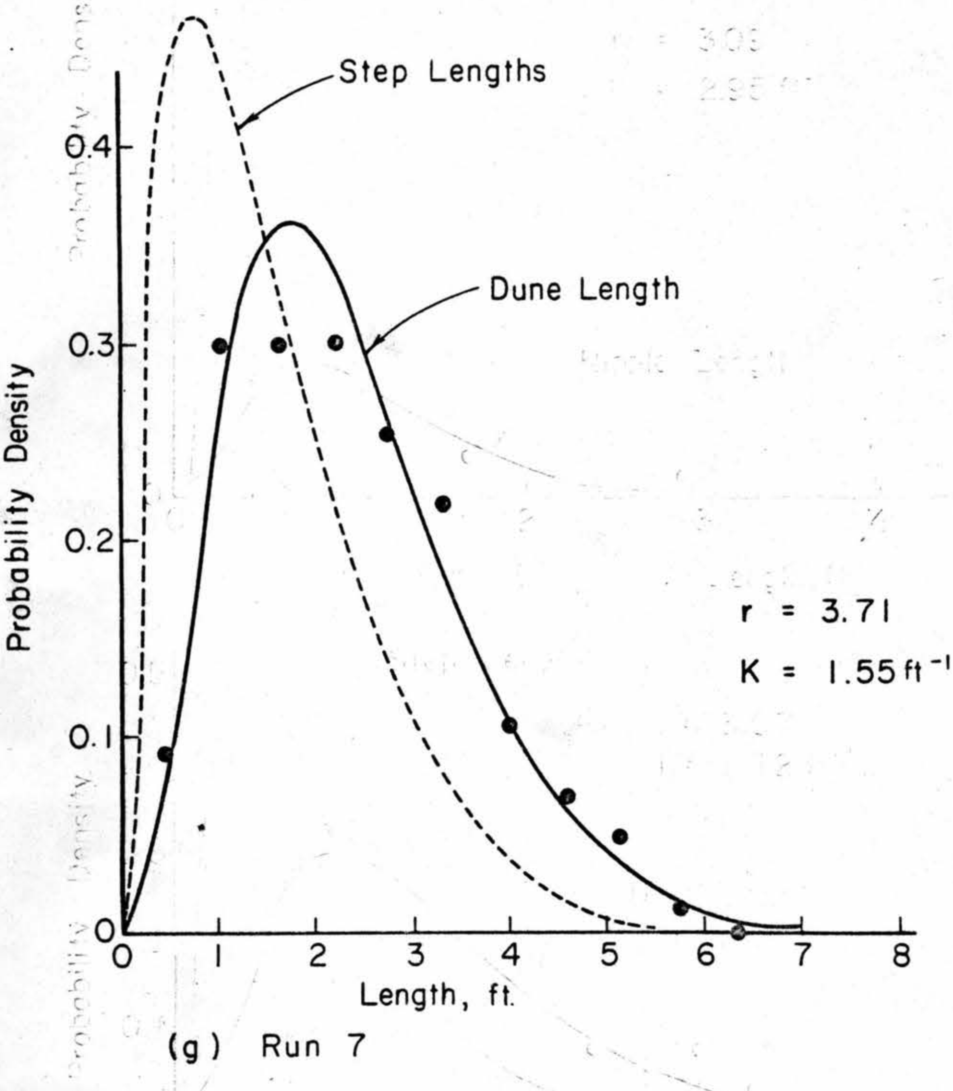


Figure 4-10. Distributions of bed form lengths with step length distributions superimposed.

with fitted gamma distributions. The step length mean and variance relate to the mean and variance of the dune lengths as shown in figures 4-11 and 4-12. There is the suggestion that the third variable of bed material size should be included in these figures. More bed material sizes should be tested to define this hypothesis. Also the flow depth could affect the relation as well.

According to the relation for the larger bed material on figure 4-11, a shorter step length corresponds to a longer dune length. One suspects that a certain size bed material would be unable to form bed features smaller than about two feet in length. Simons and Richardson (1966) report that ripples will not form in bed material larger than that with a median fall diameter of about 0.6 mm. The data presented on figure 4-11 tend to support this conclusion when the upper limit of ripple length is taken as 2 feet.

Since only two moments are required to estimate the parameters of the gamma distribution the distributions of bed form lengths can be established from easily measurable quantities. By sounding over a dune bed in a longitudinal direction, one can measure rapidly the bed profile, and with automatic data processing equipment, calculate directly the mean and variance of the dune lengths. From relations such as those in figures 4-11 and 4-12, the mean and variance of step length distributions can be determined, leading to the probability distribution of step lengths. The advantage of such an approach would be the possibility to go from an easily measured quantity directly to the distribution of step lengths. No sediment samples or other time consuming sampling methods are required. If the bed load could be

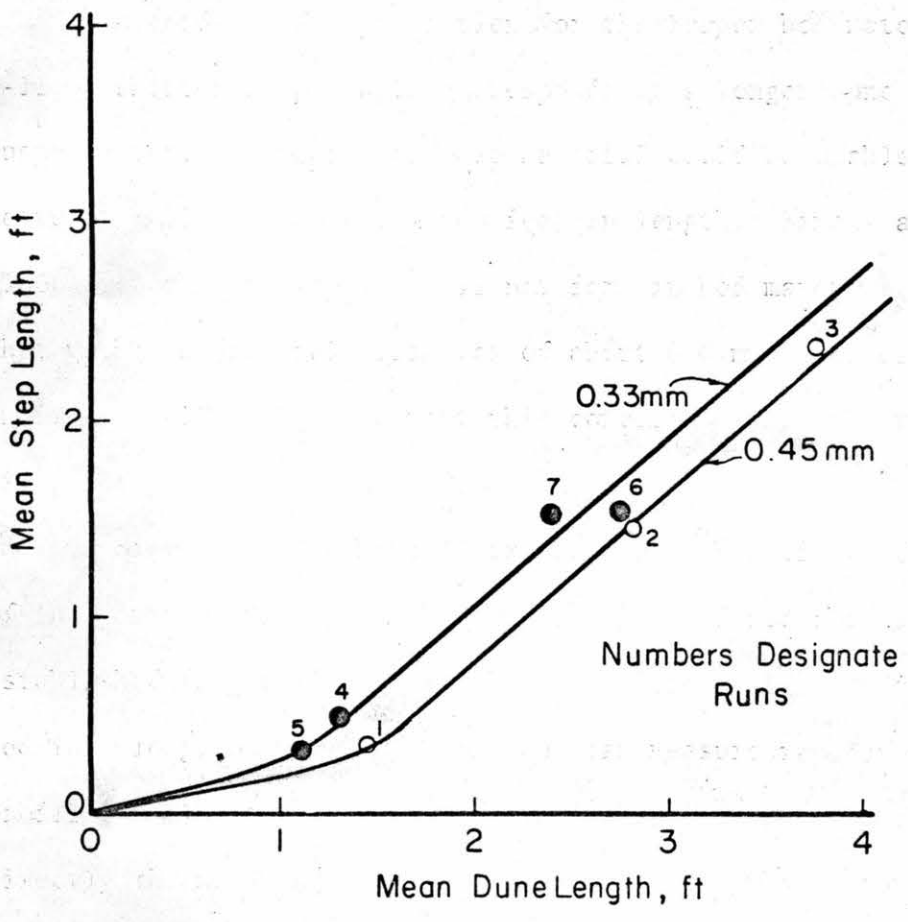


Figure 4-11. Relation between mean step length and mean dune length.

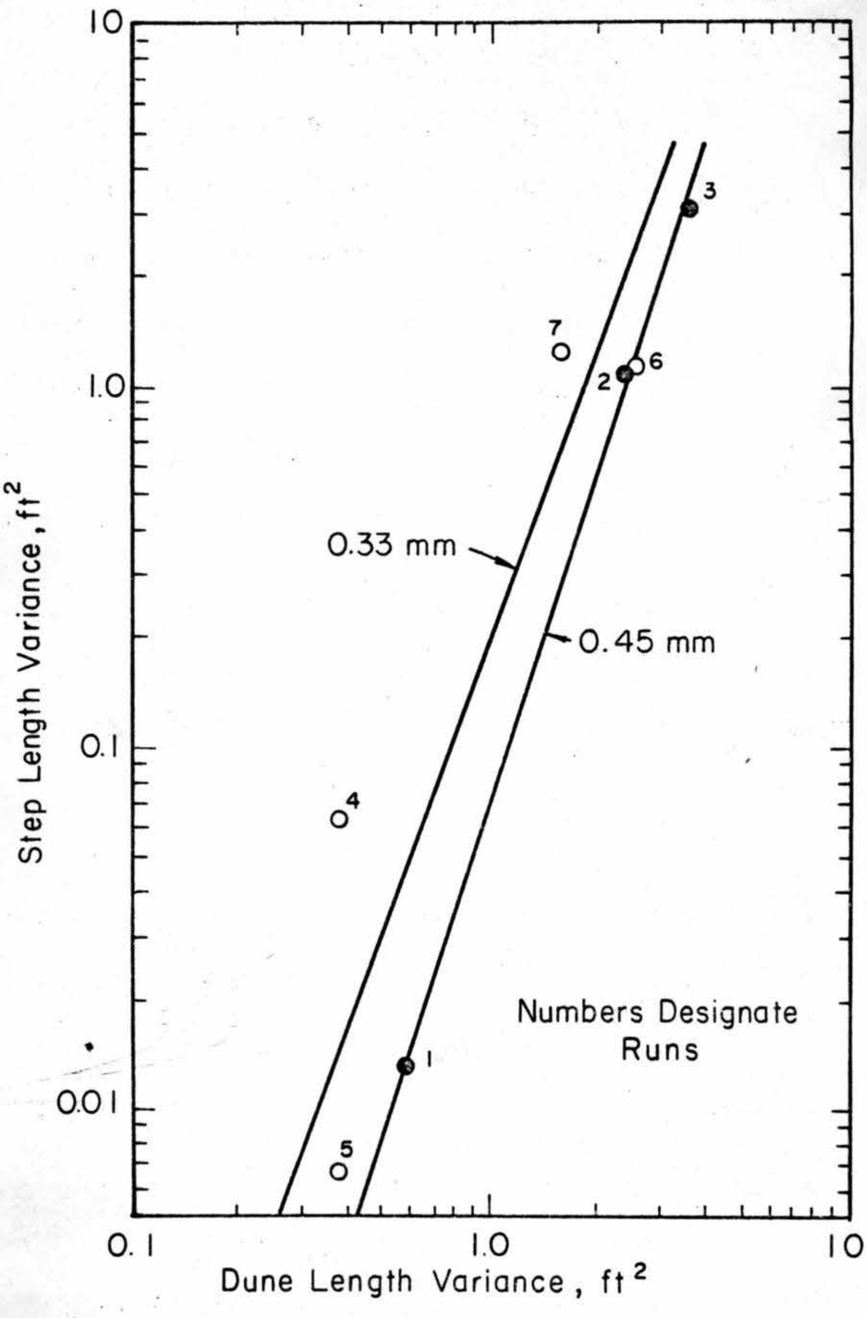


Figure 4-12. Relations between step length variance and dune length variance.

calculated from the step length properties and the suspended load from a single point sample, the total bed material load would be directly calculable from easily measured quantities.

Figures 4-13 and 4-14 show the variation of r and k, the gamma distribution parameters, with stream power for dune lengths and step lengths. The shape parameter r is approximately constant for the bed form lengths regardless of the stream power. There is a sharp delineation of the relation for step lengths indicating a difference in particle motion between ripple and dune regimes of flow. The same variability is evident in the relation for k, the scale parameter although the relation is not constant for dune lengths.

The burial definition of rest period requires that rest periods be more closely related to the motion of bed forms than to their size. An obvious measure of the motion sand waves is their celerity defined as the ratio of the mean wave length of the $y(x)$ record to the mean wave length of the $y(t)$ record. Other definitions are possible. This one is convenient because it uses easily measurable quantities.

Figure 4-15 shows the relation of sand wave celerity to stream power for the data reported by Nordin (1968) and the data reported herein. The data reported by Nordin exhibits more scatter than the other. This is attributed to the small number of observations upon which Nordin's data was based. Usually his celerity values were based on a mean $y(x)$ wave length arrived at from one flume traverse and a $y(t)$ wave length arrived at from a short duration record (less than 24 hours). The celerities reported herein are based on the average of

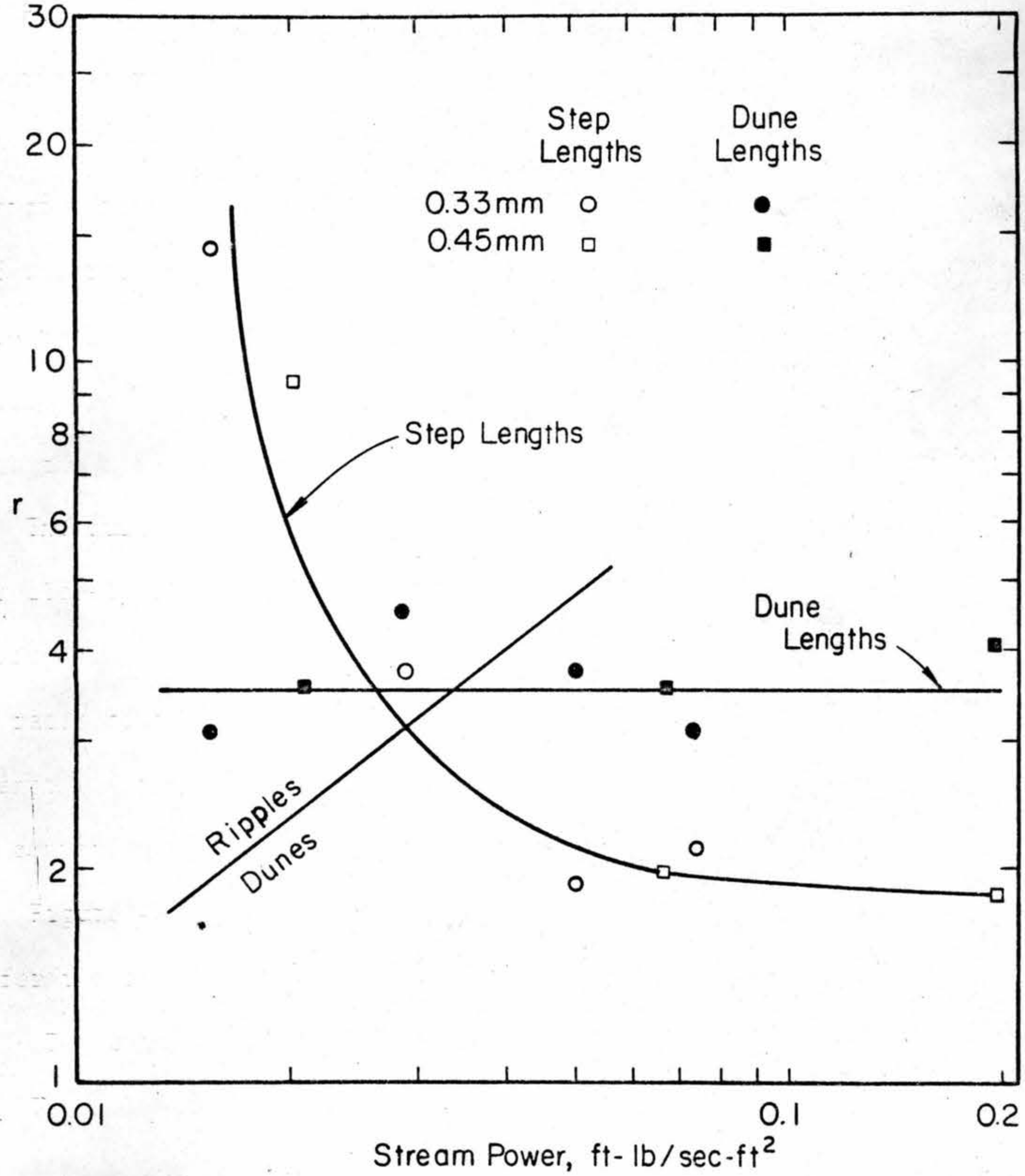


Figure 4-13. Relations between shape parameter r for dune lengths and step lengths and stream power.

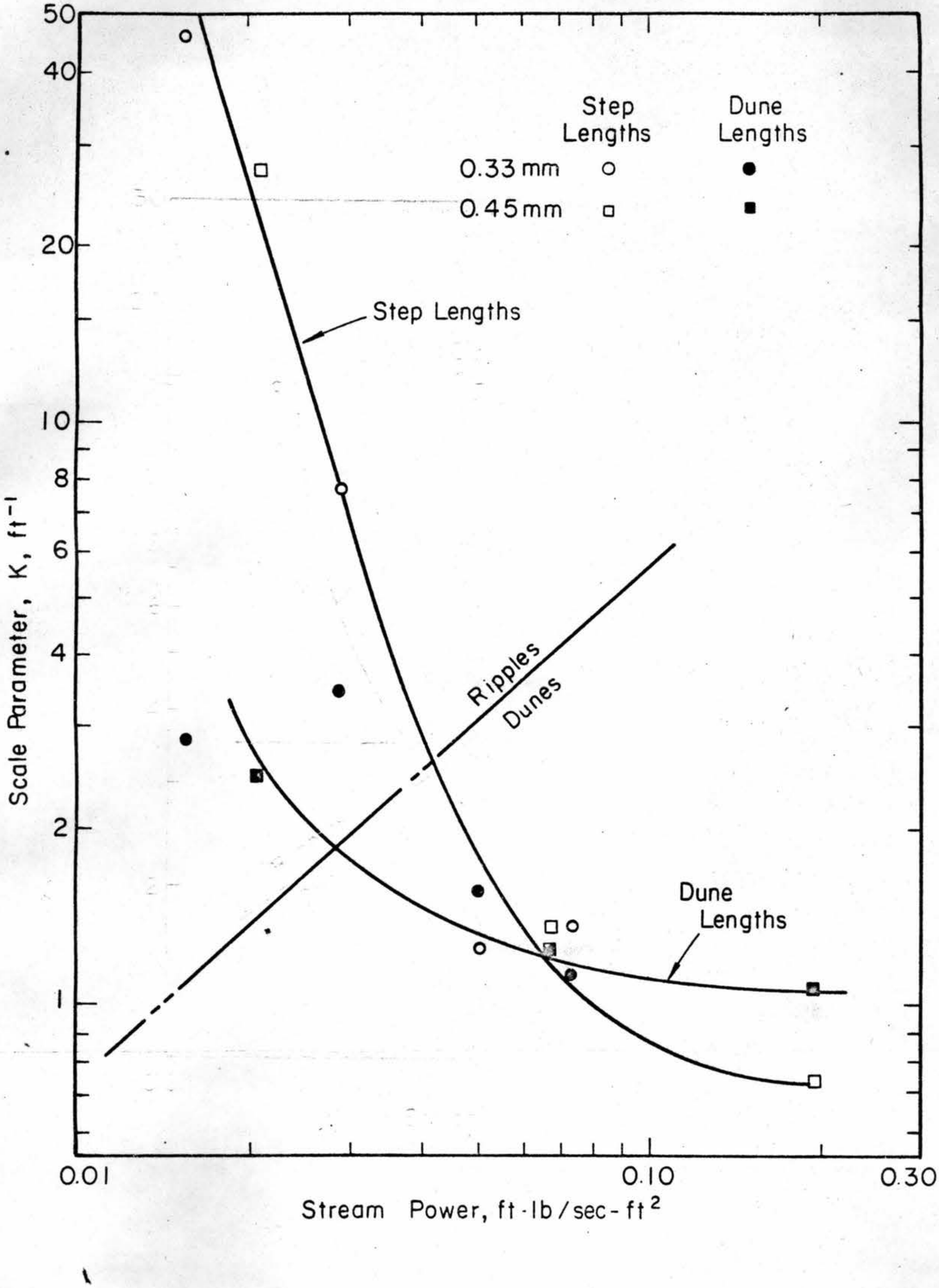


Figure 4-14. Relations between scale parameter *k* for dune lengths and step lengths and stream power.

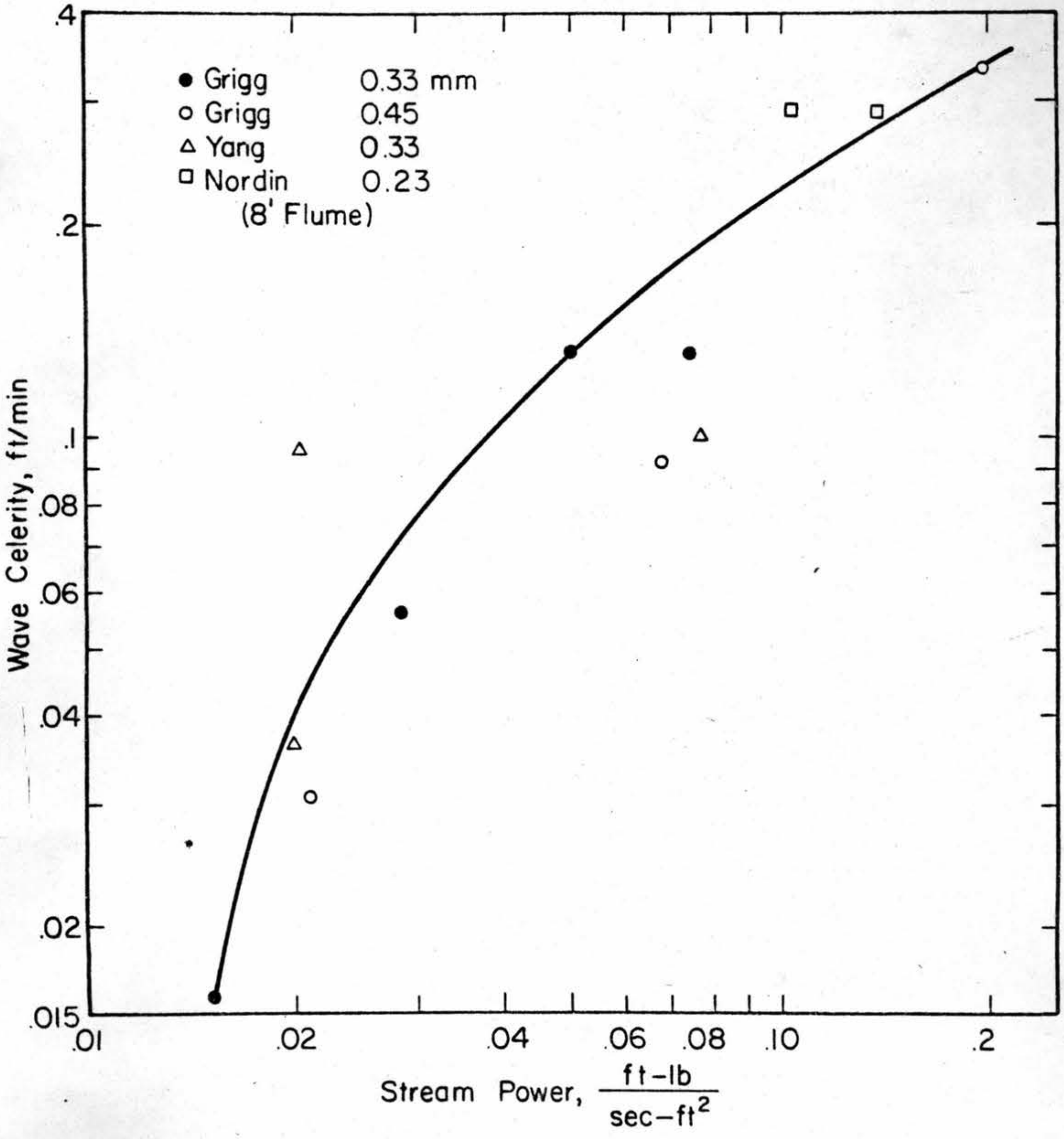


Figure 4-15. Relation between bed form celerity and stream power.

several independent $y(x)$ records and $y(t)$ records of longer duration (longer than approximately 3 days).

Figure 4-16 shows the relation between mean rest period and wave celerity. The relation is rather well defined in the dune range but exhibits scatter in the ripple range where rest periods are of quite long duration. This scatter is attributed to the limited number of observations upon which the mean rest periods are based for the higher two values. In runs 1 and 5 only 12 and 14 observations were made respectively.

The significance of figures 4-8 through 4-16 is that step length is shown to be a function of dune length and rest period duration is shown to be a function of bed form celerity. Apparently bed form size and motion can be predicted from flow and sediment properties. Thus there appears to exist the possibility of predicting particle motion from flow and sediment properties. Although the data upon which this obversation is based is limited, interesting trends are evident.

D. Determination of $f_Y(y)$

Theoretically it is possible to measure Y, the elevation of deposition. If many independent measurements could be made, the functional form of $f_Y(y)$ could be determined. Practically, such measurements were not possible in this series of experiments.

Some insight can be gained as to the form of $f_Y(y)$, however, by testing equations (2-37) and (2-38) with the experimental data and simple models of the type shown in figure 2-5. This is done by using

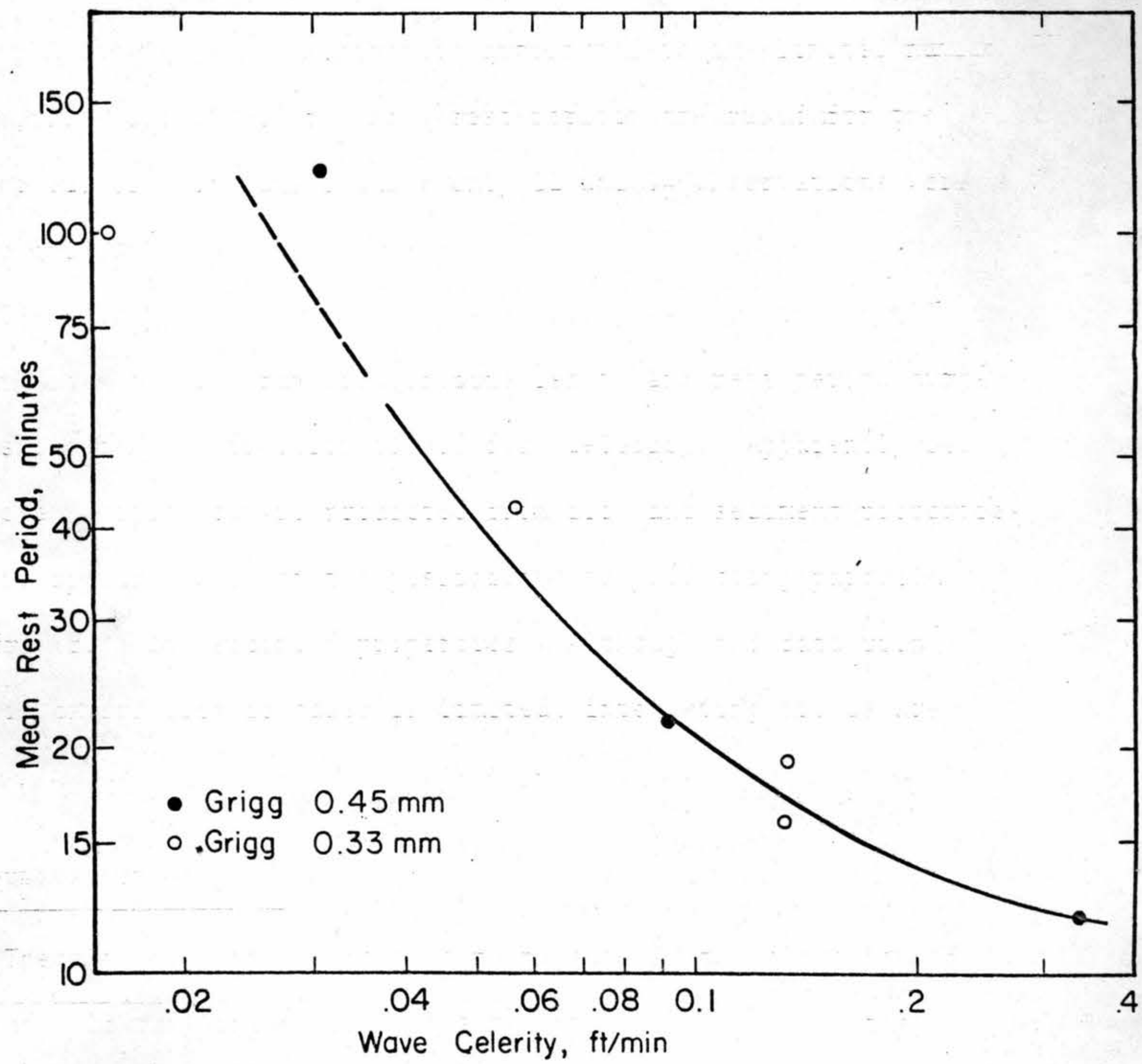


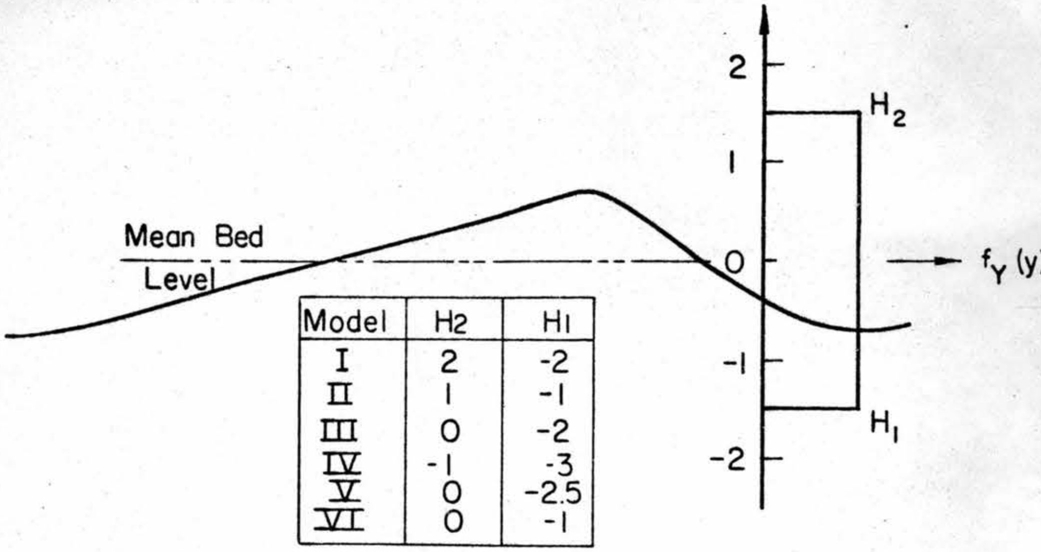
Figure 4-16. Relation between mean rest period and bed form celerity.

measured values of mean rest period, $\bar{T}$, and calculated values of $E(T)$, the expected value of the rest period.

In figure 4-18 are presented the results of calculations with six models of $f_Y(y)$. Figure 4-17 shows a typical model which is represented by a uniform distribution with varying location parameter and shape. Apparently position of the distribution is much more important than shape in the calculation of the first moment $E(T)$. Calculations were performed for runs 4 and 6 to determine what forms of $f_Y(y)$ yielded the best agreement between measured and calculated rest period.

The best agreement is with model III for run 4 and somewhere between models II and VI for run 6. This indicates that the level of the best estimate for $f_Y(y)$ varies between runs but generally lies below mean bed elevation. The measured values of mean rest period were 42.9 and 19.1 minutes for runs 4 and 6 respectively. An inspection of figures 4-6 and 4-7 shows that bed levels of -1.1 standard deviations for run 4 and -0.5 standard deviations for run 6 yield conditional rest periods approximately equal to measured mean rest periods.

Although one may not conclude what the true form of $f_Y(y)$ is from the rest period results, it is clear that particles do tend to deposit much more below the mean bed elevation than above. Consequently, deeper scour occurs below the mean bed elevation. Therefore particles will tend to move deeper and deeper in the bed to the limit of the deepest trough. This would produce a skewed distribution with the mean or centroid of tracer particles below mean bed elevation.



Model	H2	H1
I	2	-2
II	1	-1
III	0	-2
IV	-1	-3
V	0	-2.5
VI	0	-1

Figure 4-17. Evaluation of models for $f_Y(y)$.

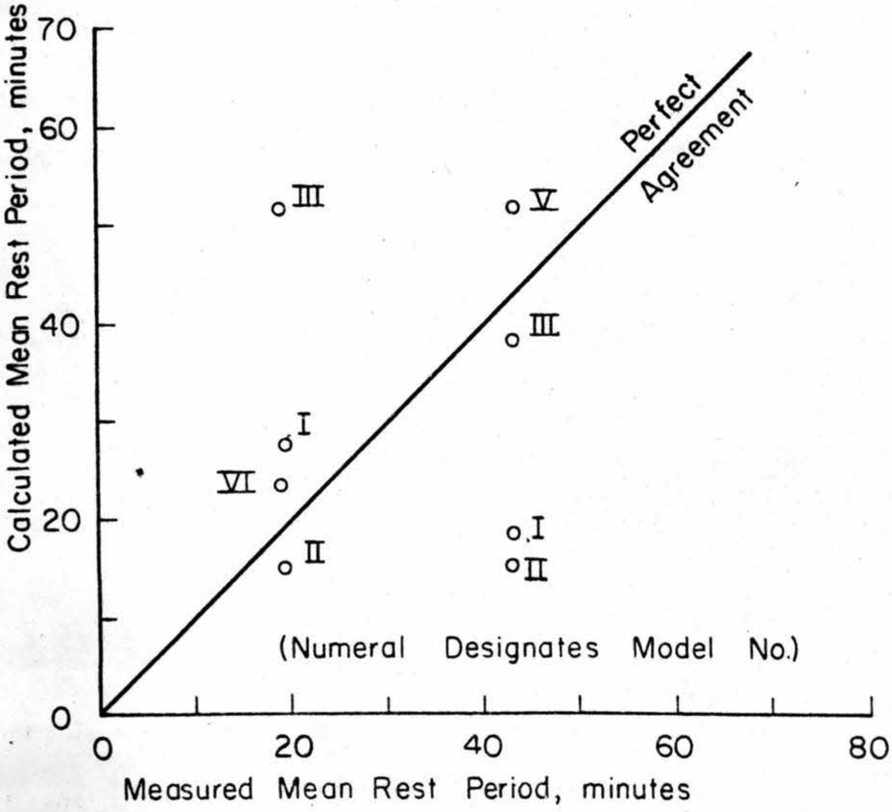


Figure 4-18. Relation between computed and measured mean rest period.

The experiment conducted in run 7 using fluorescent tracers was intended to measure how the tracers migrated vertically in the bed. No measure of dispersion as a function of time was possible since the tracers recirculated. By letting them all recirculate several times it is intuitively expected that the vertical concentration distribution would approach $f_Y(y)$.

The results of the tracer experiment are shown in figure 4-19. There are significant concentrations of tracers at 4, 5 and even 6 standard deviations below the mean and the bed elevation density has fallen to near zero beyond three standard deviations. The vertical concentration distributions for the three different sized tracers are quite similar. This could be attributed to the uniformity of the bed material.

The fact that tracers reached such low elevation is unexplained. A similar phenomenon was observed from Yang's data, some of which is presented in figures 4-20 and 4-21 for runs 1M and 2M. Direct comparison with the data shown in figure 4-19 is not possible due to a difference in flow conditions. Yang's data shows the same trends, however, with significant quantities of tracers located at points as deep and deeper than 5 standard deviations below mean bed elevation.

The presence of tracers at such great depths is strange. Only rarely does a bed form expose a point 5 standard deviations below the mean. For a Gaussian distribution the percent of time that a point of 5 standard deviations below the mean occurs is less than 0.01 percent. It is therefore quite unlikely that a point this deep would be exposed during an experiment of nominal length.

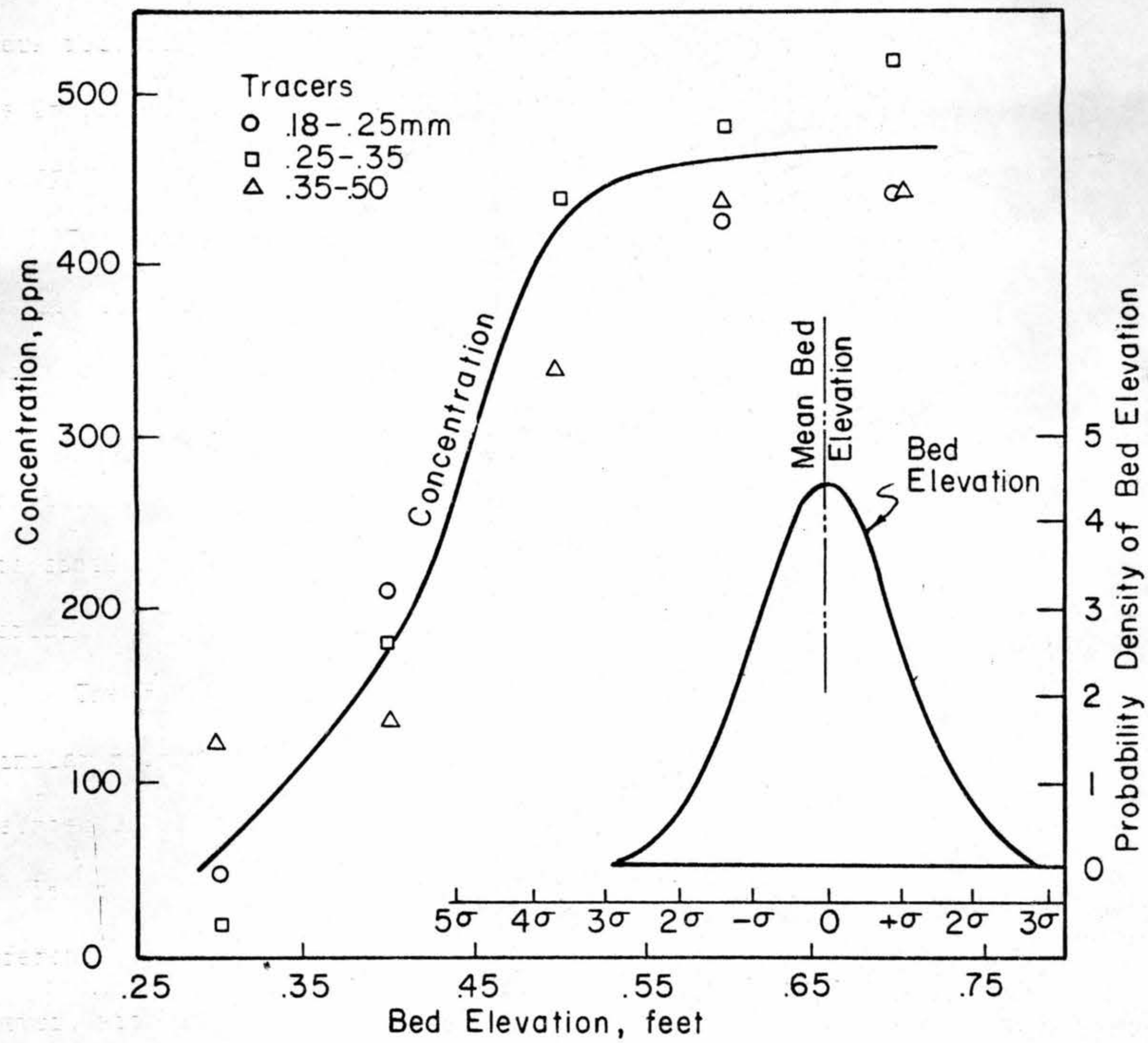


Figure 4-19. Vertical concentration distribution of tracers, Run 7.

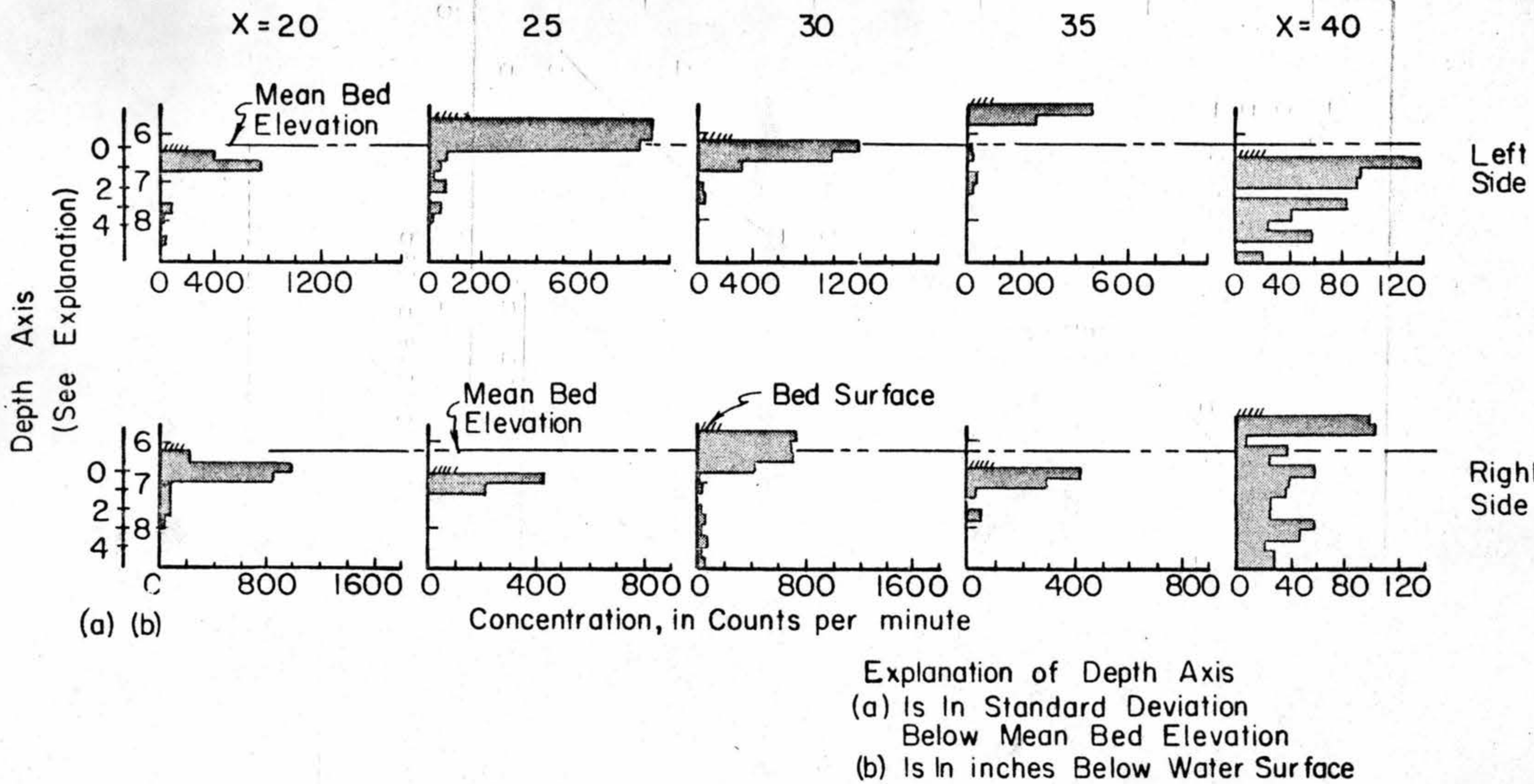


Figure 4-20. Vertical concentration distributions as seen from core sample results from Yang's Run 1M, Pass 7.

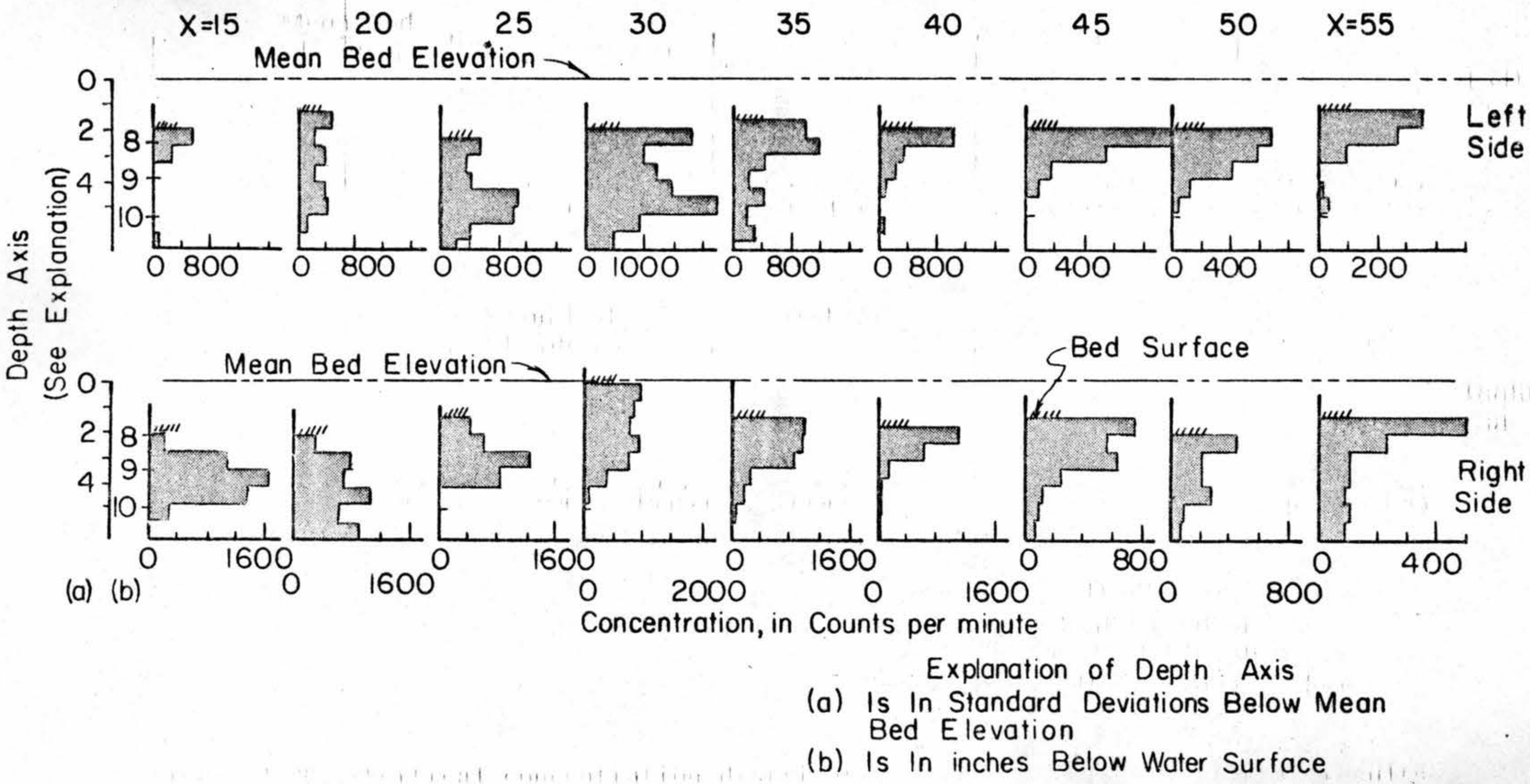


Figure 4-21. Vertical concentration distributions as seen from core sample results from Yang's Run 2M, Pass 6.

There seem to be only two possible explanations for this anomaly. Either particles can migrate downward in a bed or there is a basic error associated with the core sampling technique used by Yang and in this present study. The migration possibility appears fairly remote since many observations through glass-walled flumes have revealed no such phenomenon. There is the chance though, that wall friction could prevent this occurrence near the walls whereas it could occur in the center of the flume. One could consider that both the sand and the water are flowing and that the sand is mixing as does the water through turbulence, but through an extremely slow mechanism. The wall friction would affect the sand movement much as it does the velocity of fluid particles. The bed forms would then represent instabilities on the water-sediment interface.

The second possibility, that of an error in the core sampling, is a more likely explanation for the anomaly. The method of core sampling is to thrust a thin-walled pipe section into the medium to be sampled and to lift out the core. The forces of friction and tension serve to hold the sample in.

The core sampler used by Yang, and in the present experiments was made of 3/4-inch I. D. plastic tube with a wall thickness of 1/8 inch. It is quite possible that the sampler tended to compress the area where the sample was taken such that a lower reading was indicated for the individual cores. The standard deviations of bed elevation assumed for Yang's runs 1M and 2M are only 0.0374 and 0.0700 feet respectively. Small errors in sampling could represent large multiples of standard deviation.

In addition to a compression of the samples, some mixing may take place within a sample. It was observed by the writer that upon raising a sample to the surface, the water began to drain from the sand. The natural reaction to prevent the sand from dropping out was to rotate the sampler upward. This seemed to result in some mixing of the sample. Possibly the reason for this mixing was that a clean, uniform flume sand was being sampled. There was no fine or clay fraction to bind the core together making it one mass. Yang's sand was slightly more graded than the present sand but suffered as well a lack of binder material. Quite possibly his cores may have been mixed in the process of sampling. It has been reported that field samplings do not experience this difficulty in keeping cores in the sampler. This could be attributed to the clay binder and fines usually found in river sands. Quite possibly, the larger samplers used in the field are more efficient than the smaller ones used in the laboratory. It should be noted that the cores taken by Sayre and Hubbell (1965) do not show tracers to the great depths that the laboratory results do. Here one might suggest that the core sampling might have been more accurate, or that the fines in the bed material of the North Loup River prevented any migration downward by the tracers. Both possibilities should be acknowledged at this time.

E. Longitudinal dispersion

The two-dimensional concentration distribution function given in equation (2-7) was shown to integrate to (2-12) as the marginal case in the longitudinal direction. If one assumes the gamma and exponential distributions respectively for the step lengths and rest periods, he arrives at equation (2-15).

It appears satisfactory to use the exponential distribution rather than the gamma for rest periods since the mathematical complications introduced into equation (2-12) by the use of the gamma for rest periods do not appear warranted at this time. Generally, the chi-square test for goodness-of-fit shows the exponential to be as good as the gamma in describing rest periods.

In order to evaluate how the one-dimensional stochastic model predicts the dispersion process using the measured step length and rest period distributions, one may use the statistical properties of the distribution $f_t(x)$ as they vary with time. These properties have been given by Yang (1968) as

area under the curve $$A = 1 - e^{-k_2 t} \tag{4-13}$$

location of mean $$\bar{x} = (k_2/k_1)\ t\ r \tag{4-14}$$

variance $$s^2 = \frac{k_2 t\ r}{{k_1}^2}\ (r+1) \tag{4-15}$$

skew coefficient $$S = \frac{r+2}{\sqrt{(r+1)\ r\ k_2 t}} \tag{4-16}$$

The use of the measured constant values for r, k_1 and k_2, shows that all of these properties become functions of time.

The mean velocity of the centroid of the concentration distribution is equal to the mean velocity of the single tracer on which equation (2-15) is based. Thus equation (2-15) becomes

$$\frac{\bar{x}}{t} = \frac{\bar{X}}{\bar{T}} \tag{4-17}$$

which is true regardless of the distributions used for X and T. The distinction between $\bar{X}$ and $\bar{x}$ should be noted here. The quantity $\bar{X}$ is the mean step length whereas $\bar{x}$ is the location of the centroid of a concentration distribution.

The area under the dispersion curve is predicted to increase exponentially to the asymptote of one according to equation (4-13). Yang's data showed that the area actually decreased with time for both the dune and ripple runs. Sayre and Hubbell (1965) observed a similar decrease in area with time. The observed decreases are due to the possible loss of tracer strength with time and the increase of distance between detector and tracer.

The speed with which the tracers move is seen by equation (4-14) to be a linear function of time. Figure 4-22 shows the predicted mean distance of travel with Yang's data plotted alongside. Even though there were some differences in experimental conditions, the fit seems quite good. As seen on figure 4-22(a) the relation predicted using the parameters from the present run 4 fit Yang's run 1M (ripple) data much better than the relations predicted from either run 1 or run 5. Two conclusions may be drawn from this observation. The data from run 1 were based on the 0.45 mm bed material and tracer whereas Yang's run 1M was 0.33 mm bed material and tracers in the sieve class 0.30-0.35 mm. There apparently is a marked effect of bed material size on the speed of movement. A second conclusion that may be drawn is that the difference in water surface slope between runs 4 and 1M of from 0.00119 to 0.00088 was not very significant. The difference the other way, using runs 5 and 1M, of from 0.00069 to 0.00088 was quite significant. It was

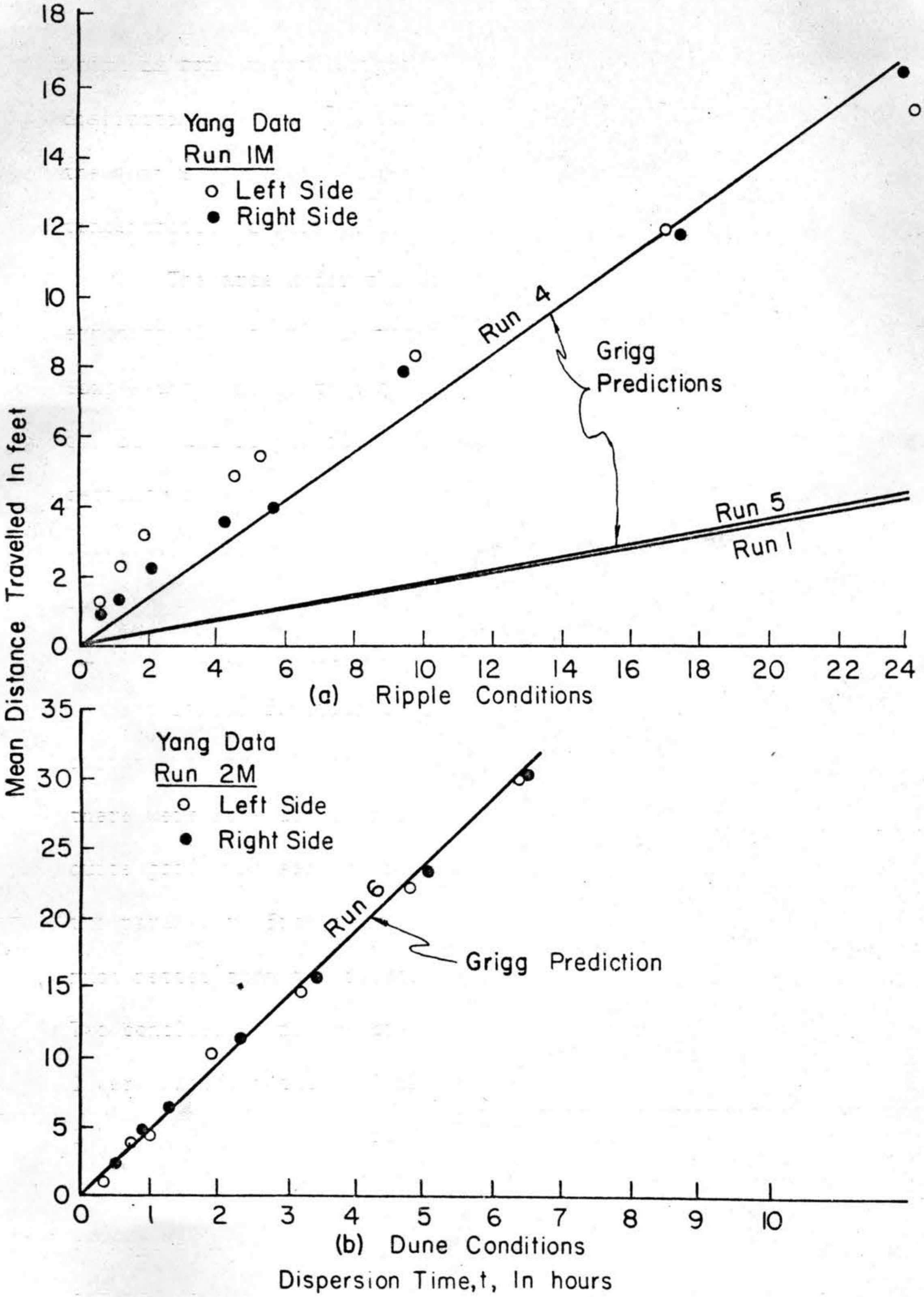


Figure 4-22. Prediction of location of mean of concentration distribution as a function of time.

evident from the $y(t)$ records taken on runs 4 and 5 that motion was considerably dampened when the water surface slope changed from 0.00119 to 0.00069. Another possibility is that Yang's water surface slope during run 1M was actually higher than 0.00088. Indeed, all the hydraulic parameters between runs 4 and 1M practically coincide except the water surface slopes.

Runs 6 and 2M were quite close in hydraulic properties. Figure 4-22(b) shows that the predicted mean distance travelled agrees quite well with Yang's data from run 2M indicating the adequacy of the general stochastic model to predict the mean distance travelled.

Equation (4-15) shows that variance should be a linear function of time. Figure 4-23 shows that Yang's experimental data do indeed follow a linear relation but that the predictions based on equation (4-15) grossly underpredict the variance, by about a factor of 4 for the ripple run and 2 for the dune run. An examination of equation (4-15) shows that an adjustment of the parameters r, k_1 and k_2 could bring the predicted variance in line with the experimental results.

Yang (1968) showed that, by assuming values of r and calculating k_1 and k_2 from equations (4-14) and (4-15) one could closely approximate a set of $f_t(x)$ curves. It is of interest to examine how such computed k_1 and k_2 values compare with those measured. Since measured values of r are available, they will be used instead of assuming values. Using this procedure on runs 1M and 2M with parameters measured during runs 4 and 6, one gets the following results:

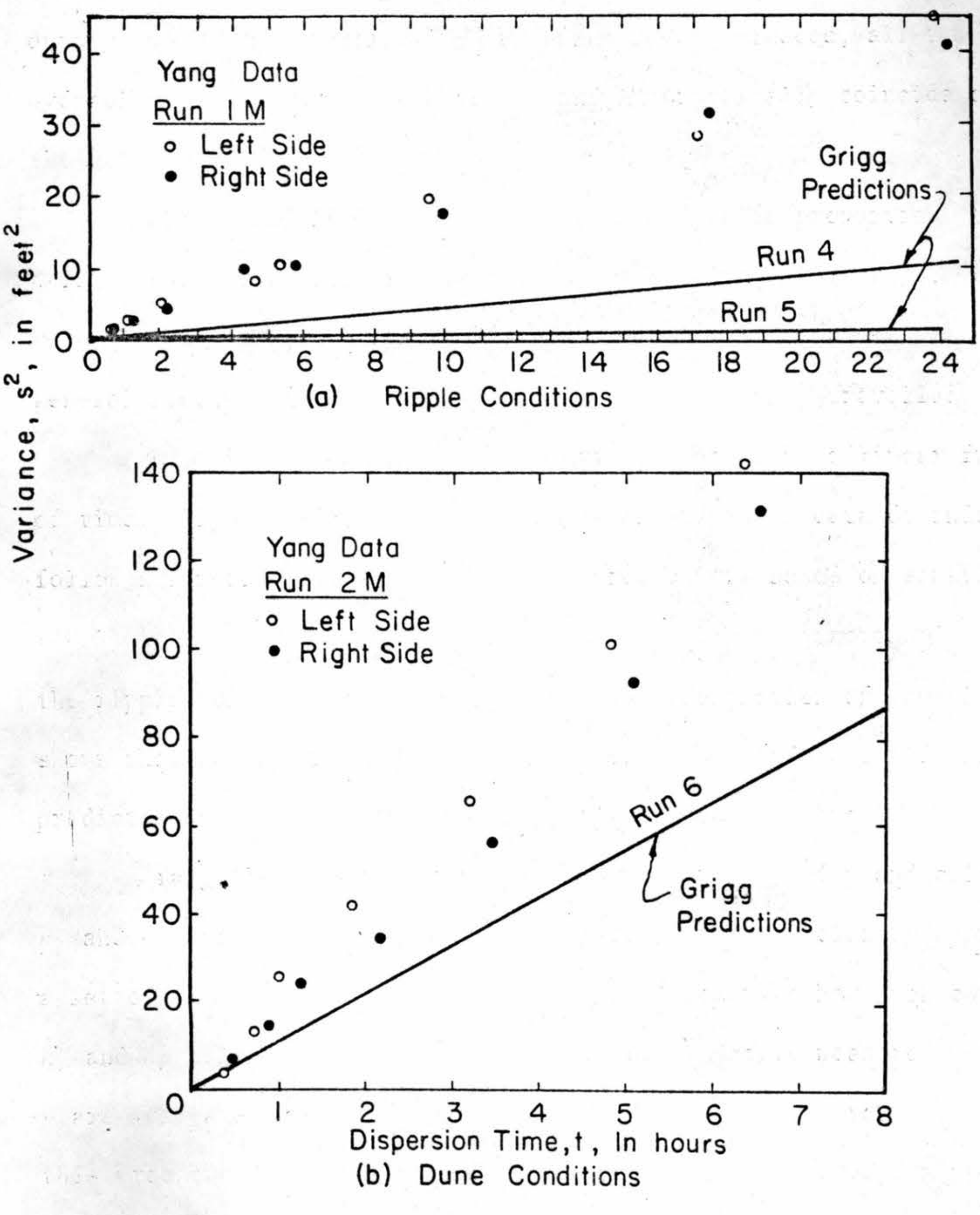


Figure 4-23. Prediction of variance as a function of dispersion time.

Run	Quantity	Yang Data	Grigg Measurement
1M	r	-----	3.76
	$k_1(ft^{-1})$	1.83	7.67
	$k_2(hr^{-1})$	.333	1.40
	$\bar{X}(ft)$	2.05	.49
	$s_x^2(ft^2)$	1.12	.0639
	$\bar{T}(hr)$	3.00	.716
2M	r	-----	2.10
	k_1	.746	1.37
	k_2	1.71	3.14
	$\bar{X}$	2.81	1.53
	s_x^2	3.77	1.11
	$\bar{T}$	.585	.267

The step lengths predicted by Yang's data are longer than those measured in the present experiments. Also, the variance of step lengths predicted by Yang's data is greater than that actually measured. The rest periods are correspondingly longer, resulting in a mean particle velocity approximately equal to that measured in the present experiments. The longer step lengths give a smaller k_1 parameter and result in greater variance according to equation (4-15).

One is tempted to explain the discrepancy in variance by the method of introducing the tracers by Yang. The methods used differed in the ripple and dune runs. In his ripple runs, Yang initially buried the tracers and allowed them to be scoured out naturally. In his dune runs he raised the tracers from the bed to introduce them essentially instantaneously into the flow. One would expect that in the dune runs perhaps there might be an initially greater than average spreading of the tracers. This does not show up in figure 4-23 as the rate of change of variance with time is essentially constant.

The discrepancy in variance cannot be explained by the definition of step length and rest period as discussed in Chapter 2. The longer step lengths and rest periods predicted by Yang's data agree better with the burial definition than with the "each movement" definition which required shorter jumps and rest periods.

According to equation (4-16) the skew parameter $S\sqrt{t}$ should be constant. Yang's data showed that the parameter generally decreases with time. Yang's flume was not really long enough to measure accurately the skewness, as the parameter involves the third moment of the distribution and the tails are of great importance in calculating the higher moments.

A possibility to explain the high variance in Yang's run 1M is that his bed may not have been in equilibrium. Were the bed aggrading, tracers could have been left behind after being covered by recirculated particles. This could result in more spread of the concentration distribution. Most probably this explains only a small part of the discrepancy in variance.

Another possibility for the large difference in variance is that successive step lengths and/or rest periods might not be independent. Note that if step lengths were not independent,

$$\mathrm{var}\left[\sum_{k=1}^{n} X_k\right] = \sum_{k=1}^{n} \mathrm{var}\,[X_k] + 2\sum_{k=1}^{n}\sum_{j=k+1}^{n} \mathrm{cov}\,[X_k, X_j] \qquad (4\text{-}18)$$

(Parzen, 1960). Depending on whether the covariance was positive or negative the variance of the sum could be larger than the sum of the variances. The process $X(t)$ might then be some sort of Markov process. There were not sufficient successive step length measurements to define well the serial correlation coefficients. Usually, the number of successive measurements was ten or less. Calculations for serial correlation for some of the longer records showed serial correlation coefficients (lag one) not significantly different from zero. These results are not definitive, however. More analyses with longer records should be performed to ascertain true independence.

F. Bed material transport

In table 3-4 are listed the measured total bed material concentrations observed during the tests. In runs 1 and 5 the sediment discharge was too small to measure accurately with the available equipment.

Figures 4-24 and 4-25 show how the measured transport data correspond to similar data taken by other investigators. Figure 4-24 is a comparison of the total load measured using the 0.45 mm bed material with the total load data given by Guy, Simons and Richardson (1966). Although the two bed materials actually differ by 0.03 mm in median diameter and in gradation from 1.41 to 1.60, the agreement is good.

Figure 4-25 shows that the data for the 0.33 mm sand agrees somewhat with that of Daranandana (1962). Yang's transport rate appears somewhat higher than that measured in this study. This may be attributed to the easily moved, flaky particles of mica present in

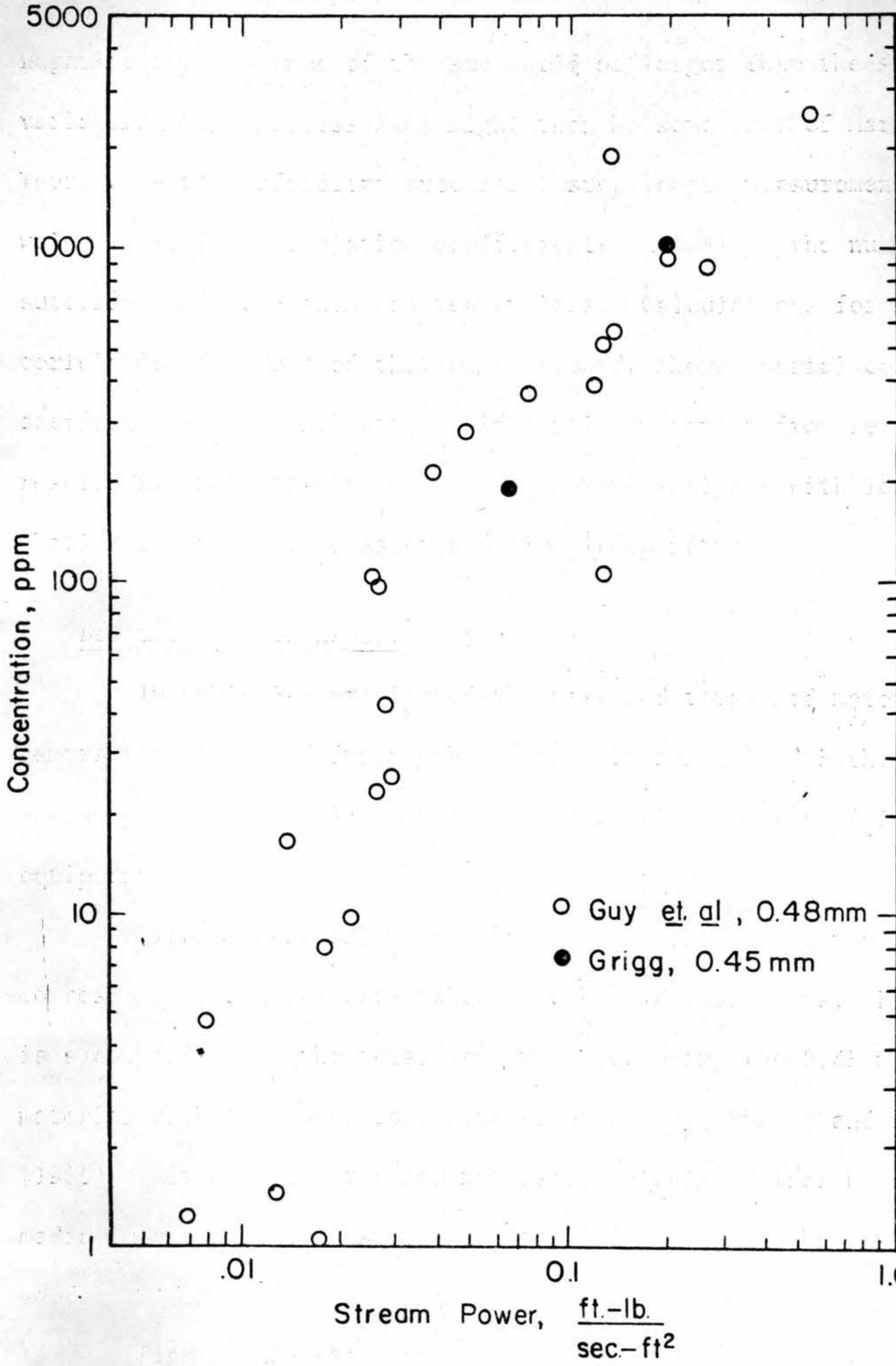


Figure 4-24. Comparison of data for total bed material transport with 0.45 mm bed material.

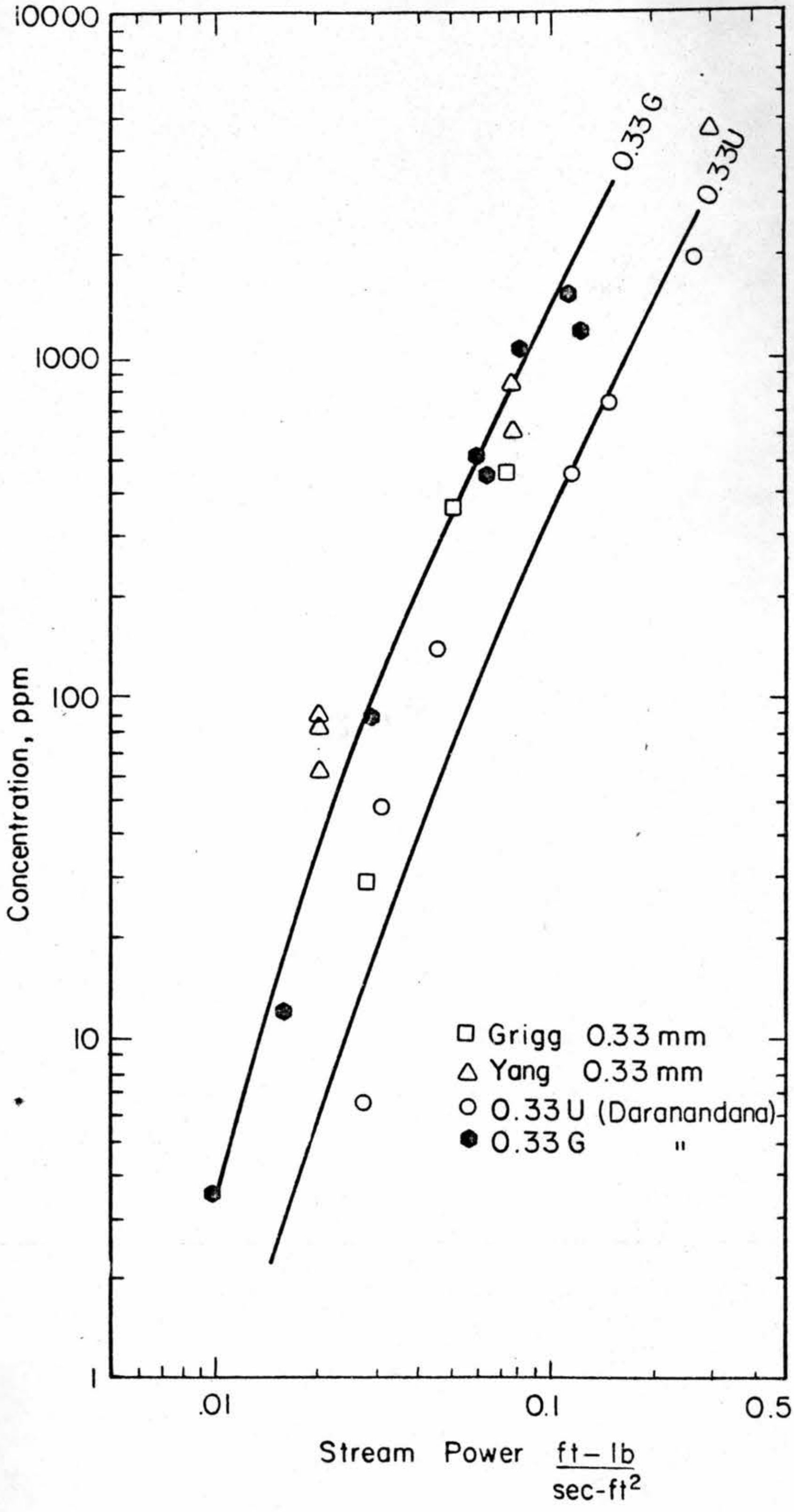


Figure 4-25. Comparison of data for total bed material transport with 0.33 mm bed material.

Yang's bed material. The data appears to fall between the curves for the uniform and graded 0.33 mm sand tested by Daranandana.

Combining equations (2-52) and (2-54) yields

$$q_s = \gamma_s(1 - \lambda)\ \delta\ \frac{k_2}{k_1} r \quad . \qquad (4\text{-}19)$$

The quantity δ in the above relation is the average thickness of the layer of movement of the bed material. This quantity was taken by Sayre and Hubbell (1965) as the average depth to which tracer particles extended below the bed surface.

From equation (4-19) one may solve for δ if, as in the present experiments, the other quantities are known. The results of such calculations for runs 4 and 6 are:

Run	$q_s\left(\frac{\text{lb}}{\text{sec-ft}}\right)$	δ (ft)	δ/σ
4	0.000994	0.0487	1.3
6	.0238	.166	2.8

The value used for porosity, λ, was 0.35.

The values of δ calculated are considerably less than the average depth to which tracers extended as given by Yang (1968). Yang's results indicate that the mean depth to which particles penetrate may increase with time.

The calculated values of δ above correspond to 1.3 and 2.8 standard deviations of bed elevation. The indication of this is that the actual average depth of movement varies with type and size of bed form. The reduced value of δ is in agreement with the indications taken from the models for $f_Y(y)$. Most of the particle movement appears to occur below the mean bed elevation. Apparently, from the transport data, most of the transport is the result of the movement of particles in a zone of varying thickness which lies below the mean bed elevation.

Chapter V

SUMMARY AND CONCLUSIONS

One of the main objectives of the study was the determination of the probability density functions needed in Sayre's general two-dimensional stochastic model. The model, given by equation (2-7),

$$f_t(x,y) = \sum_{n=1}^{\infty} f_X^{(n)}(x)\ f_Y(y) \int_0^t f_T^{(n)}(t')\int_{t-t'}^{\infty} f_{T|Y}(\tau|y)\ d\tau\ dt' \qquad (2\text{-}7)$$

is a general representation of the motion of a single particle or of the development of the concentration distribution of a group of particles. If the model is correct it will predict the motion of any sediment particle of interest, provided the appropriate parameters are known.

The model was tested using Yang's dispersion data. It was able to predict the properties of the dispersion process with limited success. Specifically, it predicted the mean velocity of the centroid of the concentration distribution but failed to predict the rate of spreading, or variance, of the distribution. The reasons for the failure are not known but it is suspected that the assumptions made in deriving the model are too restrictive.

The study has established basic forms for the probability distributions needed in Sayre's model. Also, relations have been determined between flow properties and the parameters of the distributions. From the relations presented herein, one can determine particle motion from flow properties under conditions similar to those prevailing during the flume experiments. More experimental data is required to extend the relations to the general case. Enough information has been gathered, however, to suggest the directions in which future investigations should proceed. The remainder of this chapter is devoted to discussion of specific items about which such information has been gathered or questions have been raised in the course of the study.

It is known that the geometry of dunes is dependent on depth of flow, diameter of bed material and slope of energy gradient. The same statement is true of ripples with the exception that the geometry appears relatively independent of depth. In this present study it is shown that a relation exists between step length and dune length. One should, therefore, be able to predict step length from a knowledge of the length of dunes, an easily measurable quantity.

No doubt the variable of gradation should be included with those affecting dune geometry. In this study, and in others, it was noted that bed forms changed at different points on the stream energy spectrum depending on the bed material gradation. Perhaps the inclusion of bed material gradation would help solve the bed form prediction problem.

The gamma distribution appeared to fit very well the data for step lengths. The gamma is a good distribution to work with for this type of data since its shape is quite flexible. Aside from being a good empirical fit to the data, there is no real theory to predict the form of the step length distribution. It is interesting to note that both the dune length and step length follow the gamma distribution. As seen in figures 4-13 and 4-14, the parameters of the distribution for dune lengths are less sensitive to changes in flow intensity than those for step lengths. This phenomenon demonstrates clearly that the mechanism of bed form formation and particle movement changes with flow.

conditions. One sees immediately the futility of trying to express particle motion in terms of anything but statistics.

The distinction between ripples and dunes noted by Simons and Richardson (1966) shows up clearly in figures 4-13 and 4-14. The shape parameter r describing the distribution of step lengths varies sharply with stream power in the ripple range. In the dune range r is nearly constant for the step lengths. The parameter r for bed form lengths is nearly constant throughout the spectrum of stream power. Similar results are noted for the scale parameter k, although not to the extent noted for r.

The conclusion evident is that there is a difference in particle motion between ripples and dunes. Particles move over a ripple bed in response to increasing fluid shear and velocity whereas the particle motion over a dune bed is affected more by length of bed form.

When the properties of bed features are described by the mean values of many observations there is good correlation with stream power. In particular, the mean length and height of the features correlates well with stream power within limitations. There appears to be a good possibility that rest periods and step lengths can be accurately estimated from easily measurable bed properties.

The distributions of unconditional rest periods seem to follow the exponential distribution which is a special case of the gamma distribution with fixed shape parameter. In some cases the gamma distribution with shape parameter slightly less than unity appeared to fit the data a bit better, but the improvement did not warrant the additional complications involved.

Actually the exponential distribution for rest periods should be modified so that no zero duration values are possible. This could be accomplished by a transformation of coordinates to allow some minimum duration rest period. Such a transformation would be mathematically satisfying but practically speaking unnecessary since the concentration distribution functions are not valid at the origin.

The mean and variance of the conditional rest period durations appeared to relate extremely well to bed elevation. The conditional rest period durations were assumed to equal the lengths of upward excursions above the level H of the process $y(t)$ as measured by a stationary bed level sounder.

Because the mean and variance of a sample completely determine a gamma distribution, it is possible to write an expression for the conditional rest period density in a closed form. If the exponential distribution is used, the measured density function becomes almost identical to one assumed earlier by Mielke as given in equation (2-41).

By using expected values as shown in equations (2-37) and (2-38) the measured values for $f_T(t)$ and $f_{T|Y}(t|y)$ can be used to gain some knowledge of the density function $f_Y(y)$ for the elevations of deposition which has, as yet, not been measured. In reality, a knowledge of $f_Y(y)$ is quite basic to an understanding of the erosion-deposition process as well as the formation and migration of bed forms.

In this study some revealing observations were apparent concerning the form of $f_Y(y)$. According to the data, most of the deposition occurs below the mean bed elevation. According to core sample data taken by Yang and earlier by Sayre, the mean position of tracers was generally well below mean bed level. Core sample data taken in this present study indicated similar results to an even greater extent.

The accuracy of core sample data is suspicious due to sampling methods. However, the comparison of measured rest periods with the integral of the conditional rest periods showed a similar requirement for $f_Y(y)$ to be skewed below the mean bed elevation. The rest period data did not require particles to be deposited to the great depths exhibited by the core sample data. The data is not sufficient to define the form of $f_Y(y)$ but adequate to show the depositional trends. A requirement certainly exists to reexamine the initially assumed depositional models.

Yang's general one-dimensional stochastic model was adequate to describe the mean velocity of the concentration distribution, particularly for the dune run. Yang's data indicated little difference in the mean velocity of coarse or medium tracers over a dune bed. This observation is consistent with the present conclusion that mean step length and rest period are dependent more on bed form characteristics than on particle size. This conclusion is conditioned on the requirement that the particles must be moving as bed load and not in suspension. The difference in size between Yang's coarse and medium tracers was not sufficient to show in general that grains migrate at the same mean velocity regardless of their size. No doubt similar experiments with very fine and very coarse tracers could define the difference better. Such a difference must exist to explain the downstream sorting of sediments by size.

Yang's data for movement over a ripple bed is rather inconclusive, particularly because the coarse tracers moved faster than did the medium tracers. One explanation for this could be the extreme sensitivity of particle motion to hydraulic conditions in the ripple range. This sensitivity was noted by the writer during the course of his experiments and is particularly evident in the steep curve of figure 4-13.

The discrepancies noted in the predicted and measured variances of the concentration distributions are evidence that further work is necessary on dispersion models. They point out that at the present time, emphasis should be placed on the models of particle movement as well as on the forms of the step length and rest period distributions. The forms of these distributions should be investigated further to determine how they vary with hydraulic conditions.

Several explanations for the discrepancy in variance are possible. Points that warrant further investigation include the assumption that successive step lengths and rest periods are independent and the anomaly observed that particles tend to be deposited quite deep in the bed. It was observed that transport occurred in a rather thin layer of bed material but deposition occurred to great depths. Quite possibly more tracers are buried and left behind than the stochastic model predicts.

The picture of sand transport which emerges from this study is one of a complicated process, as yet unyielding to analysis by simple models. In some respects the models considered herein have achieved success in describing the process. For example, the good agreement between the rate of movement of a single particle and of a concentration distribution is encouraging. Other facts, however, point to the presently unsolved facets of the transport. Happily, the indications are consistent.

There is no satisfactory explanation for the presence of tracer particles at such great depths in the bed. The comparison of rest periods as measured directly and as calculated from bed form data requires, however, that deposition occur mostly at low points in the bed. A question thus arises as to the detailed nature of the erosion-deposition process which places particles at such low points in the bed.

The transport rates measured suggest that movement does not occur uniformly in a thick layer equal to the mean depth of penetration of tracer particles below the mean bed elevation; rather transport occurs in a layer of thickness depending on the type of bed form and intensity of shear. The relation of step length to dune length shows as well how motion changes with bed shear. Particles begin by jumping a fraction of a dune length. As the stream power increases, the particles jump greater distances and the distribution of step lengths becomes more nearly like that of dune lengths. One suspects that in the transition range from dunes to plane bed, the distributions might become the same.

As has been noted before, the motion of fluvial sediments changes in response to many factors. The influence of some of these factors on the motion of sediment particles has been studied here. A better understanding of the transport process, as well as some direction for future studies is the indicated result. With better information on the motion of each particle in a bed, the gross transport and dispersion process should be better understood.

Suggestions for future research

The indications of this study reveal several areas of promise for future research. Some of these areas are as listed below:

1. Probability distributions for step lengths, dune lengths and rest periods should be more thoroughly investigated. Sufficient data are reported herein to investigate the fit of distributions other than those used. In particular, the distributions for conditional rest periods show promise for more intense investigations.

2. The influence of hydraulic and sediment parameters on the mean and variance of step lengths should be more thoroughly investigated. In particular, the parameters of bed material size and gradation and of depth of flow should be studied. It is suggested that the study of single particle motion may lead to a better understanding of the formation and movement of bed forms and the associated transport and resistance to flow.

3. Experiments should be designed to study the vertical movement of tracers in a bed. Many questions concerning the erosion-deposition process basic to loose-boundary hydraulics depend for their answers on how particles move vertically in a bed. Indications of this study are that greater mixing of grains occurs than heretofore was suspected. There is a dearth of good reliable data to substantiate the extent and/or existence of this mixing.

4. The motion of particles of greatly different sizes and specific gravities over a sand bed should be studied. A better understanding of the hydraulic sorting process would result from such investigations.

5. An attempt should be made to include bed material gradation in relations for prediction of bed form. It is suggested that bed form is almost entirely determined by stream power, fall diameter and gradation of the bed material for a flow of constant depth. The inclusion of depth of flow in the relation should solve the problem of prediction of bed form.

6. More data should be gathered on the lengths and heights of bed features such that definitive relations can be established between these dimensions, flow conditions and sediment properties. The use of the sonic sounder and computer promises to remove much of the scatter from such relations.

7. The assumptions made in the development of Sayre's general two-dimensional stochastic model should be examined to determine if they are overly restrictive. An improved model should be developed that will predict more accurately the detailed development of the dispersion process.

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REFERENCES

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APPENDIX

STEP LENGTH AND REST PERIOD DATA

STEP LENGTHS AND REST PERIODS

Step lengths, *X*, are reported in feet and rest periods, *T*, in minutes. Both quantities are reported in series of uninterrupted sequences. A step length on a certain line followed the rest period which is reported on the same line.

T	*X*	*T*	*X*	*T*	*X*
Run 1, particle 2		Run 2, particle 2		Run 2, particle 2	
268	0.4			(continued)	
46	.3	14	1.7	19	2.0
193	.6	14	2.5	19	2.1
67	.1	14	1.5	5	
36	.3	7	1.2		
32	.5	4	1.0	7	1.7
25	.4	7	.9	7	.6
464	.5	7	1.4	8	.7
94	.3	14	1.1	8	.8
93	.4			8	1.2
15	.3	60	2.2	15	2.0
147	.3	7	.3	6	.8
		7	1.1	2	.8
Run 2, particle 2		11	.8	5	.6
4	2.0	4	.3	2	1.2
17	1.2			47	2.7
4	.4		1.1	17	1.7
4	.6	35	.5	26	2.4
119	2.2	3	.9	112	1.7
52	1.1	16	3.3		
10	.7	5	.5	4	2.0
7	4.6			11	.4
30	1.8	1	.3	47	1.5
5	1.8	3	.7	4	2.4
44	1.2	39	2.0	3	1.5
8	3.6	4	.8	5	.4
341		8	.7	2	1.1
		2	1.2	5	.5
27	4.0	5	1.3	9	1.2
6	1.0	12	1.3	9	1.0
7	.3	2	1.0		

T	*X*	*T*	*X*	*T*	*X*
Run 2, particle 2		Run 2, particle 2		Run 2, particle 7	
		(continued)			
8	1.6	14	0.9	5	1.3
23	1.9	14	1.3	3	1.1
9	.7	21	2.2		
		19	1.2	13	1.4
24	1.1	8	.9	58	1.0
7	.2	14	1.7	2	1.6
	5.4	31	3.4	19	1.8
		100	.3	4	1.0
	1.4	7	4.2	11	1.5
		7	.3	15	1.4
	.9	30	2.2		
4	.9	7	.4		2.7
7	1.3	7	1.7	38	2.9
6	.7			6	1.1
4	.6	Run 2, particle 7			
25	2.0	17	1.4	7	2.1
7	.4	4	1.5	1	.4
7	.5	4	.6		
7	.5	21	2.1	1	.9
4	1.0	5	4.0	6	.7
4				8	.5
		5	.7	7	1.8
23	.7	5	1.1	24	1.2
31	5.1	12	1.4	5	1.5
10	1.9	13	1.2	2	.9
132	2.5	7	1.2	25	1.5
9	2.1	6	.9	4	1.6
23	2.2	13	1.4	3	.1
22	4.1	7	1.2	11	1.9
61	3.3	6	.4	17	.9
	1.5	7	.8	30	1.2
		131	1.1	66	
	2.0	6	.8		
20	1.4	79	2.5	Run 3, particle 2	
13	.7	7	1.3	3	1.3
		6	.8	21	2.1
	2.5	7	1.1	4	2.5
13	1.6	5	.8	24	1.3
14	1.7	9	.9	4	.3
88	.6	7	1.3	7	2.2
143	.9	12	1.1	84	1.8
7	.5	7	1.3	73	2.4
7	.3	13	2.0	7	5.1
		489	3.2		
		7	.6		

T	X	T	X	T	X
Run 3, particle 2 (continued)		Run 3, particle 2		Run 3, particle 2 (continued)	
7	2.8		0.8	7	1.5
7	1.9	16	3.6	10	.8
7	1.9			7	3.4
7	1.5	3		14	3.0
17	2.4			14	1.1
		15	3.2	76	4.7
8	17.9	8	1.5	21	4.4
15	2.5	7	2.5	4	1.6
7	2.7	7	2.5	3	2.8
	1.8	8	1.1	4	1.0
		8	.9	7	1.0
4	3.3	4	1.6	7	3.2
3	1.0	12	2.0	7	4.0
4	3.5	140	.9		.8
3	1.7	11	1.5		
4	2.1	1	.3	4	4.5
2	3.6	1	1.4	13	.4
8	.5	2	1.6	13	2.2
14	5.1	7	2.8	3	1.8
4	2.9	7	2.6	71	.8
3	.7	7	2.5	17	.9
14	1.3	7	2.3	3	2.8
11	4.0	51	1.5	3	1.2
27	4.0	35	.6	18	.5
7	3.8		5.5	4	1.7
	1.5			3	2.8
		7	2.7	40	1.0
3	1.1	7	1.9	4	2.2
7	2.2	13	3.9	14	1.5
4	2.1	11	2.5	7	3.3
3	.6	7	3.8	7	2.5
	2.6	3	2.2	7	.1
		4	1.4		2.5
	3.8	3	.8		
4	3.1	3	2.6	7	3.1
24	.9	6	1.6	3	3.3
7	2.7	8	2.4	4	2.5
1	2.3	6	2.2	3	.7
4	2.5	72	1.6	4	4.1
	3.1	1		2	.3
				1	1.9
	1.3	14	.3	21	2.2
7	.7	18	1.4	7	1.5
		3	2.7	7	1.7

T	*X*
Run 3, particle 2 (continued)	
7	2.1
7	2.7
16	2.5
4	1.0
	2.3
3	.9
17	1.2
24	2.0
27	2.2
4	2.4
1	1.5
1	1.2
2	7.0
4	1.4
17	2.2
10	2.9
4	2.7
	3.5
	1.7
14	3.5
4	1.6
7	5.0
	3.5
14	1.3
	4.2
7	2.9
7	1.7
1	2.1
7	1.6
7	3.2
7	2.3
10	1.7
10	2.7
24	3.2
7	4.7
13	5.0
	2.6
Run 3, particle 2	
	2.0
3	1.6
14	2.9
7	3.0
4	2.1
3	.8
4	.8
39	5.0
14	.9
9	4.5
50	.6
	8.3
Run 3, particle 7	
4	3.1
7	1.6
3	2.3
7	2.0
10	3.8
7	1.9
7	.6
14	3.1
7	2.3
7	2.8
7	2.2
7	2.1
7	1.7
	.8
4	5.5
2	1.8
28	2.8
32	2.5
14	1.9
15	1.7
7	1.2
7	3.9
20	2.4
27	5.0
20	6.0
10	.5
14	3.1
5	3.5
Run 3, particle 7 (continued)	
1	8.1
1	3.9
8	.9
4	1.3
7	2.1
7	3.0
3	2.0
18	.7
14	5.0
	2.6
4	3.6
3	1.0
7	3.1
2	.7
	1.2
	3.8
5	2.5
4	.3
	1.6
	7.6
	1.0
10	1.7
13	5.4
7	4.1
3	3.7
50	5.2
4	3.9
18	1.1
3	8.1
	1.6
	2.4
3	5.7
18	4.9
10	2.0
4	18.9
	1.4

T	*X*	*T*	*X*	*T*	*X*
Run 3, particle 7		Run 3, particle 7		Run 3, particle 7 (continued)	
4	2.2	2	4.8	7	0.4
3	1.1	4	1.1	4	1.9
14	1.6	7	1.8	7	1.1
27	1.5	3	3.1	7	2.3
4	2.2	7	.3	14	1.3
3	1.6	4	1.1	17	6.6
7	1.6	3	2.1		3.9
4	1.1	4	1.4		
52	3.0	7	.6	14	9.6
5	3.9	7	2.3	17	7.6
1	.4	7	2.0	3	2.6
4	1.4	3	.9	35	2.7
17	.6	61	2.0	17	10.4
4	2.3		2.8		3.6
7	1.4				
7	2.8	4	2.1	7	1.0
	1.5	3	2.1	7	1.5
		14	1.6	7	1.7
3	.6	4	1.7	4	1.8
7	2.2	3	1.7	10	3.8
4	2.5	14	2.8	4	2.5
3	1.4	71	1.6	17	6.1
4	8.8	4	2.6	10	2.8
3	1.1	7	1.6	34	2.5
7	1.4	3	1.1		11.4
28	2.2	7	1.6		
14	3.2	3	1.2	4	2.8
14	3.8	7	.3	3	2.3
12	3.8	7	2.4		.8
	2.2	3	1.5		
		4	1.6	Run 4, particle 3	
7	2.8	41	2.6	14	0.3
7	2.3		5.5	24	.4
4	4.7			81	.4
3	1.4	7	3.5		.5
11	3.0	14	3.3		
1	2.1	7	2.4		.6
7	1.6	4	1.1	36	.8
19	.7	3	1.4	7	.6
4	2.1	4	3.0	23	.2
3	.8	3	2.1	28	.7
6	1.0	36	.9	7	.4
	1.4	3	3.6	42	.5

T	*X*
Run 4, particle 3 (continued)	
14	0.3
4	.4
59	.4
7	.5
7	.7
7	.5
21	.9
	.3
54	.2
7	.8
23	.2
47	.9
20	.4
20	.5
57	.8
33	.5
10	.6
80	.4
14	1.7
6	.4
25	.2
13	.7
10	.4
17	.3
70	.5
	.5
Run 4, particle 6	
7	.5
20	.5
15	.3
35	.3
20	.2
57	.7
13	.4
12	.3
14	.4
7	.3
157	1.1
50	.1
12	.3
	.2
	1.1
	.8
Run 4, particle 7	
14	0.3
49	.4
21	.7
141	.5
14	.7
51	.3
10	1.0
92	.6
7	.3
	.5
7	.5
7	.4
7	.3
64	.3
14	.6
57	.3
246	.5
104	.5
27	.3
5	.4
12	.4
13	.2
40	1.1
313	.6
235	.2
10	.5
16	.4
14	.5
10	.3
37	1.0
54	.9
7	.5
7	.5
182	.4
13	.2
27	.5
33	.5
40	.5
13	.4
39	.2
23	.4
3	.8
7	.8
410	.2
43	.6
Run 4, particle 7 (continued)	
7	0.4
149	.2
36	.2
30	.5
13	.5
Run 5, particle 3	
48	0.3
56	.3
20	.3
47	.4
57	.4
323	.3
Run 5, particle 6	
96	0.5
24	.3
48	.3
332	.3
Run 5, particle 7	
39	0.5
106	.3
170	.2
10	.3
Run 6, particle 3	
38	3.1
3	1.2
9	.9
7	1.5
26	1.0
13	1.6
18	2.4
106	2.5
13	6.4
39	.9
7	1.1
13	1.3
6	1.0
33	.8
	2.0
	1.7

T	X
Run 6, particle 3	
7	1.0
10	.7
10	1.7
4	.9
3	5.5
3	1.0
52	.5
7	1.3
20	1.0
3	1.0
7	.7
3	.9
27	1.6
4	1.0
3	1.2
51	.8
	5.9
	.8
	2.9
7	2.6
20	1.9
7	2.7
29	1.6
6	.2
4	.4
3	.6
4	1.0
6	1.2
7	.6
7	1.8
43	1.3
2	2.2
7	1.0
6	.4
7	.9
11	.7
4	2.6
3	1.2
3	3.0
4	1.6
17	1.4
42	2.8
7	2.5
	3.4

T	X
Run 6, particle 3	
13	1.3
7	1.6
7	1.4
34	3.7
7	.3
10	3.1
3	1.2
7	1.5
3	.7
10	.8
4	.6
7	2.9
3	1.2
3	1.0
39	.7
5	.3
7	.2
7	.7
6	.9
10	1.5
24	1.3
10	.2
4	2.7
3	.9
24	.6
	3.2
7	.9
21	.8
183	1.7
7	1.8
21	.3
7	1.9
29	.9
9	2.0
10	.9
4	.9
23	1.0
28	3.2
3	1.4
4	1.0
27	1.8
3	1.1
7	2.8
7	2.2
13	2.1
	1.3

T	X
Run 6, particle 3	
129	0.3
3	1.2
7	2.2
14	2.4
7	1.4
	.5
28	1.9
14	1.6
7	1.8
19	2.3
7	1.4
7	1.8
3	.5
3	2.3
49	1.4
16	2.0
4	1.4
11	1.5
24	1.9
3	1.2
6	3.0
8	3.2
7	.8
17	1.0
4	.4
56	.8
	2.0
68	.6
7	1.3
45	.4
31	3.0
36	2.4
3	.7
10	.6
3	5.7
15	.7
27	1.7
184	5.3
3	.4
8	1.1
7	1.1
7	.9
10	1.0
4	.6

T	X
Run 6, particle 3 (continued)	
7	1.0
3	1.4
7	2.0
238	1.2
Run 6, particle 6	
4	3.8
3	.4
97	2.0
44	.6
3	3.3
4	2.4
40	1.9
7	3.1
43	1.5
7	1.0
3	2.0
16	1.0
4	1.2
6	2.0
7	.6
3	.8
4	.9
7	1.1
6	.3
7	1.7
3	.1
4	.5
6	1.5
7	2.5
7	
	2.8
4	3.0
3	1.3
31	.8
	2.5
59	1.1
3	.8
35	1.9
20	2.7
4	2.0
4	1.2

T	X
Run 6, particle 6 (continued)	
25	0.6
6	3.0
3	1.4
47	1.0
3	1.9
10	1.1
7	1.1
7	1.6
3	1.4
4	.7
41	.8
7	1.4
14	2.2
4	1.4
3	.8
3	1.2
	.5
	4.5
7	1.8
7	.7
3	.9
3	.7
49	.4
7	1.9
6	.4
3	5.6
7	1.0
77	1.2
35	2.3
6	.6
4	.9
20	1.0
3	3.3
32	.3
7	2.0
5	.7
33	1.7
40	.3
9	1.7
10	.8
4	.5
7	.6

T	X
Run 6, particle 6 (continued)	
179	0.8
14	4.6
21	.2
14	4.4
7	2.3
7	.2
	2.1
Run 6, particle 7	
8	0.3
70	.8
103	1.6
6	1.1
18	.8
3	.7
7	.6
1077	.7
207	1.9
156	5.5
7	1.0
7	2.5
4	.5
3	2.0
29	.8
3	1.6
15	1.6
17	2.2
11	2.0
7	1.1
9	.8
48	.7
70	.5
7	1.7
8	
13	1.1
3	1.7
31	.3
3	.7
10	1.8
31	2.0
3	.6
7	2.0
	1.2

T	*X*	*T*	*X*	*T*	*X*
Run 6, particle 7		Run 7, particle 3		Run 7, particle 3	
	3.1		0.6		1.5
		7	.4	7	2.2
	2.3	7	1.8	45	
63	1.3	10	3.1		
69	5.1	7	.9		1.1
8	2.6	7	.7	7	5.6
17	1.8	10	1.6	7	1.0
12	1.6	7	1.1	7	1.3
6	1.7	4	1.1	3	.7
34	.9	3	1.0	10	.9
24	1.9	31	.9	3	.8
3	2.0	43	2.8	5	.8
3	.8	22	1.3	3	2.8
7	1.0	3	1.5	10	1.9
7	.5	3	1.5	18	2.1
6	2.2	21	1.8	3	1.1
41	2.8	7	1.0	30	.9
19	1.0	13		33	.6
7	.6			4	.9
20	2.7		1.1	3	1.1
20	3.4	34	1.1	4	.5
	1.5	7	2.0	20	1.2
		37	.8	3	1.1
7	.7	44	.8	3	.5
6	3.0	7	1.6	3	1.4
6	2.3	16	1.0	4	2.5
33	1.4	3	.8	17	.9
17	2.1	3	.7	7	2.3
10	2.2	6	.9	13	1.5
10	1.4	15	1.3	7	.5
129	1.9	48	.4	7	1.4
		17	2.8	20	.7
Run 7, particle 3		20	2.6	3	
	2.2	4	1.3	3	
3	.8	6	1.5		
4	.8	4	2.4		.7
353	1.7	3	1.0	121	2.5
7	1.0	7	.9	7	1.8
16	1.2	14	2.3	14	1.2
36	1.2	3	.5	7	.9
27	2.1	4	.5	8	.9
6	1.4	6	2.0	21	1.6
16	1.3	4	.5	4	.9
13	1.5		1.1	4	1.0
13	1.8			14	1.6
				6	1.6

T	*X*	*T*	*X*	*T*	*X*
Run 7, particle 6		Run 7, particle 6		Run 7, particle 6	
	1.9			(continued)	
11	.2		0.6	7	1.1
7	1.0	7	1.3	4	.7
71	.5	7	.5	3	4.9
3	.9	13	.3	14	2.5
3	.5	14	1.5	4	
14	1.2	14	1.6	19	
36	2.8	7	.8		
3	1.1	3	1.1		1.1
12	1.1	4	1.0	13	1.4
7	.7	6	1.2	23	1.3
15	.7	4	.6	13	2.5
3	.8	10	1.1	8	.9
14	2.0	4	3.8	11	9.2
3	1.0	3	.8	3	1.0
7	1.6	13	4.2	33	2.2
9	1.9	4	3.0	41	2.2
30	2.0	3	.5	6	.8
3	1.0	99	1.2	8	1.0
3	.5	7	3.0	10	2.2
4	.4	3	1.2	18	2.7
3	1.6	4	2.0	4	1.3
11	.7	7	2.0	3	1.5
3	1.4	7	1.0	175	1.3
7	.3	41	.6	6	1.8
48	.4	7	1.4	8	1.6
20	.4	7	1.3	91	2.3
3	1.5	4		5	
73	1.9				
21	1.2		.7	Run 7, particle 7	
3	3.3	7	1.3		1.0
3	.7	72	1.3	44	2.1
	.7	17	1.7	14	1.6
		7	1.5		
	1.5	21	2.9		2.3
3	.6	23	2.0	36	1.4
4	.9	24	1.7	3	1.0
15	4.3	9	1.3	4	.9
7	3.0	32	1.6	3	1.2
3	.5	7	2.6	4	1.1
4	.6	21	2.4	3	1.1
8	.6	14	2.3	10	1.3
		21	2.5	7	.7
		39	3.0	7	1.8
		13	1.3		

T	*X*	*T*	*X*	*T*	*X*
Run 7, particle 7 (continued)		Run 7, particle 7 (continued)		Run 7, particle 7 (continued)	
5	2.7	4	0.2	10	2.7
6	.8	17	2.1	7	1.7
3	2.8	20	3.1	31	3.5
41	.4	7	1.5	13	4.5
4	1.0	7	.4	4	1.3
3	.2	40	1.1		1.9
7	.8				
3	.6		.5		2.4
3	.9	27	3.0		3.5
35	1.3	27	1.9		2.0
17	1.8	54	10.4	8	1.8
7	1.1		1.0	14	1.8
42	1.3			22	2.6
13	3.2		1.1	35	1.0
7	.7	14	1.2	18	2.7
7	2.2	7	1.6	14	2.0
7	.4	14	1.2	21	2.7
		21	1.3	14	2.2
	1.3	7	1.6	7	2.1
4	1.1	14	.8	11	3.7
3	1.2	77	1.0	11	.6
41	2.5	6	1.5	21	1.8
20	2.3	8	1.8	8	1.0
				6	.9