DISSERTATION

EEG SUBSPACE ANALYSIS AND CLASSIFICATION USING PRINCIPAL ANGLES FOR BRAIN-COMPUTER INTERFACES

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Abstract

EEG Subspace Analysis and Classification Using Principal Angles for Brain-Computer Interfaces

Brain-Computer Interfaces (BCIs) help paralyzed people who have lost some or all of their ability to communicate and control the outside environment from loss of voluntary muscle control. Most BCIs are based on the classification of multichannel electroencephalography (EEG) signals recorded from users as they respond to external stimuli or perform various mental activities. The classification process is fraught with difficulties caused by electrical noise, signal artifacts, and nonstationarity. One approach to reducing the effects of similar difficulties in other domains is the use of principal angles between subspaces, which has been applied mostly to video sequences.

This dissertation studies and examines different ideas using principal angles and subspaces concepts. It introduces a novel mathematical approach for comparing sets of EEG signals for use in new BCI technology. The success of the presented results show that principal angles are also a useful approach to the classification of EEG signals that are recorded during a BCI typing application. In this application, the appearance of a subject's desired letter is detected by identifying a P300-wave within a one-second window of EEG following the flash of a letter. Smoothing the signals before using them is the only preprocessing step that was implemented in this study. The smoothing process based on minimizing the second derivative in time is implemented to increase the classification accuracy instead of using the bandpass filter that relies on assumptions on the frequency content of EEG. This study examines four different ways of removing outliers that are based on the principal angles and shows that the outlier removal methods did not help in the presented situations.

One of the concepts that this dissertation focused on is the effect of the number of trials on the classification accuracies. The achievement of the good classification results by using a small number of trials starting from two trials only, should make this approach more appropriate for online BCI applications.

In order to understand and test how EEG signals are different from one subject to another, different users are tested in this dissertation, some with motor impairments. Furthermore, the concept of transferring information between subjects is examined by training the approach on one subject and testing it on the other subject using the training subject's EEG subspaces to classify the testing subject's trials.

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CHAPTER 1

Introduction

Brain Computer Interfaces (BCIs) are communication and control systems that are used to translate brain signals into commands and messages in order to control applications such as typing letters using a virtual keyboard, moving a pointer on a computer display, and turning on or turning off the lights [1, 6]. BCIs help paralyzed people who lost their ability to communicate and control the outside environment since they cannot use their normal voluntary muscles. BCIs help those people who suffer from one of the neurodegenerative diseases such as cerebral palsy, amyotrophic lateral sclerosis, or multiple sclerosis (MS). As in any communication system, BCIs consist of an input part, which is the brain signals using EEG electrodes, either invasive or noninvasive, and an output part, which is the command to communicate with the external devices. Current noninvasive BCIs are categorized into different groups based on the type of the electrophysiological signals that are used to control and guide the BCI operations. One of these groups uses P300, which is an event related potential (ERP) that measures the brain response to a stimulus. These BCIs are based on the classification of multichannel EEG signals recorded from users as they respond to external stimuli or perform various mental activities. The classification process is fraught with difficulties caused by electrical noise, signal artifacts, nonstationarity, and a lack of understanding about how EEG signals vary for different mental tasks. Current BCI methods lack reliability for these reasons.

The P300 wave is a relatively large positive deflection in the voltage that starts about 300 milliseconds after the target stimulus is presented to the user [3]. It is usually produced when a subject is presented with a rare but expected stimulus, which is called the oddball paradigm [7]. Each stimulus and the following EEG signals are referred to as trials. Trials containing the expected stimulus are called target trials, and other trials are nontarget trials. It may be possible for P300-based BCIs to be used directly by a user without any training, since it is a natural response for a desired selection. However, P300 amplitude and latency is different between users and varies with a subject's fatigue level [8]; current P300 classification methods do require some training.

Recent work in subspace analysis, including discriminative ways of determining principal angles between subspaces, have the potential for dealing with noise and nonstationarity in EEG. To date they have been applied mostly to video sequences to improve the classification accuracy. In image recognition, there has been a lot of work on considering one image at a time for object recognition. Recently, techniques based on explicit image set matching have been developed in order to improve the robustness and the classification accuracy [9]. In the face recognition field, using image sets becomes more important than using a single snapshot, especially with the availability of modern cheap devices for capturing video streams. In addition, by using sets and sequences of images, more information about the variations in the appearance of the target subjects, such as varying illuminations, expressions, and movements can be provided. It has been shown that high classification accuracy can be reached through modeling image sets via subspaces and calculating the principal angles or distances between these subspaces to recognize the similarities and differences between them [10].

1.1. The Dissertation Objective

In this dissertation, the subspace representations of EEG used in BCI applications will be studied and the principal angles will be measured between theses subspaces. These principal angles between subspaces of EEG trials are used to detect the presence of P300s, detect and remove outliers, and classify the EEG trials.

In this study, we hypothesize that with this technique the EEG P300 trials will be classified more accurately even with the artifacts and noise in the signals, specially compared to other algorithm, including Linear Logistic Regression. Comparison of subspaces by principal angles allows classification to ignore natural variations and capture difference between P300 and non-P300 trials.

The natural variation of the human brain, which is one of the limitations of EEG measurements, makes the classification process harder and not accurate. In the experiment showed here, an offline analysis of EEG signals recorded from different subjects are tested and different protocols are examined in order to evaluate the presented method.

As mentioned above, individual trials of EEG are often very noisy, making it difficult to detect a P300 wave. The effects of noise on P300 data is somewhat alleviated by smoothing in time and averaging over multiple trials. For BCI applications, averaging over multiple trials increases the time required for the user to make a selection and so it is to be avoided. However, smoothing can be performed on individual trials. In this experiments, data was smoothed using Stickels [11] algorithm which approximates the data by minimizing a squared error and a roughness measure defined by the second derivative of the approximation.

One of the concepts that are focused on in this dissertation is the transferring of information between subjects. This point is studied by training the method on one subject and testing the method on the other subject in order to classify the P300 trials using the training subject's subspaces. Furthermore, the principal angles between the training subspace and the testing subject subspaces will be measured. In addition, different numbers of trials are used to create the training and testing subspaces in order to examine the effect of the subspace size on the presented method. Moreover, removing the principal angles outliers using different calculations is tested to check if this step enhances the classification accuracy.

1.2. The Dissertation Contributions

This work contributes to the Brain Computer Interface field because it is the first method that uses the principal angles between subspaces to reject the outliers and classify the EEG signals. In this dissertation, principal angles are calculated and used for P300 classification. Classifying the EEG signals is not an easy task and it is a big challenge to find a good classification method that gives high classification accuracy.

In this dissertation, different experiments and scenarios were tested to examine the presented method. The highest accuracy that has been obtained in the first experiment is 90% for two subjects, while the average accuracy of the 4 subjects is 82.75%. In the second experiments, the best accuracy was 97% for one of the subjects and the average is 85.5%. Transferring between subjects is one of the scenarios that was tested in this dissertation. The highest accuracy that was obtained is 87% when testing on one of the subjects; the average across testing on different 4 subjects is 76%. These results are reached using a 1 to 4 trials in each subspace. The state-of-the-art in P300 classification achieves 60 to 90 % using average of 10 to 15 trials [12].

In the following chapters, results in detail will be described. Briefly, the presented P300 classifier method gives high accuracy compared with other classifiers. In addition, removing the outliers did not increase the classification accuracy in the presented experiments. Creating the training subspaces using a small number of trials adds an advantage to this approach compared with the other methods that require more trials.

1.3. Overview

The dissertation is organized as follows. Chapter 2 gives the background concepts to help in understanding the presented problem and methods. In addition, a summary of the principal angle approaches that were implemented in image recognition and EEG fields are discussed. In Chapter 3, the proposed method will be explained in detail. Chapter 4 discusses the results that have been obtained after applying the proposed method on a P300-based BCI application, as well as the comparison with other classification method such as Linear Logistic Regression and other recording system such as Biosemi Active Two. The work is concluded in Chapter 5 and the future work is also outlined in this chapter.

CHAPTER 2

Background

In this chapter, some of the background concepts and information are presented in order to better understand the problem and the proposed methods that will be discussed later in this dissertation. Furthermore, current methods using principal angles between subspaces in image processing and in EEG signals classification will be reviewed. In addition, the limitations of these methods will be discussed and presented.

2.1. Brain-Computer Interfaces (BCIs)

Brain Computer Interfaces (BCIs) are communication and control systems that are used to translate brain signals into commands and messages in order to control applications such as typing letters using a virtual keyboard, moving the wheelchair, and turning on or turning off the lights [1]. BCIs are designed for paralyzed people who cannot use or depend on the brain's normal output pathways of peripheral nerves and muscles to communicate with the external environment. As in any communication system, BCIs consist of an input part, which is the brain signals using EEG electrodes, either invasive or noninvasive, and an output part, which is the command to communicate with the external devices (see Figure 2.1). The EEG electrodes are located on the scalp based on the international 10-20 system as shown in Figure 2.2. Even numbers relate to the electrodes on the right hemisphere, while the odd numbers relate to the electrodes on the left side. The letters refer to the lobe such as frontal, temporal, central, parietal, and occipital lobes [2]. Many features can be considered in the BCI design such as amplitude values of EEG signals, Power Spectral Density (PSD) values, AutoRegressive (AR) parameter, Adaptive AutoRegressive (AAR) parameter, and Time-frequency features [13]. These features are noisy, non-stationarity, and

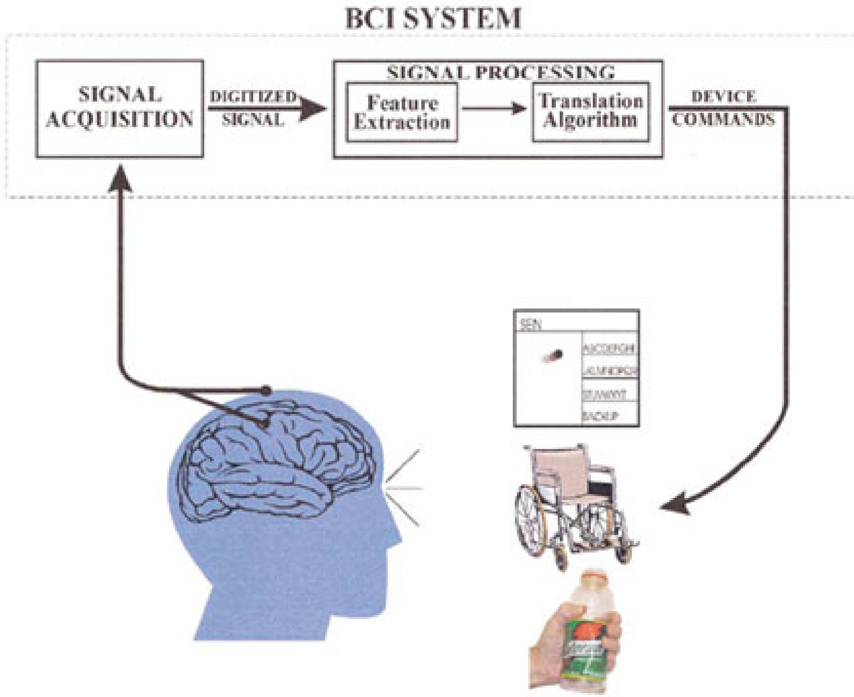


FIGURE 2.1. Basic design and operation of any BCI system [1].

high dimensional since they are extracted from several channels at several time segments and have time information because the brain patterns are related to the specific time variation of the EEG.

Current BCI are categorized into five groups based on the type of electrophysiological signals that are used to control and guide the BCI operations. One group uses visual evoked potentials (VEP) that are based on the direction of the eye gaze to the visual stimulus that has a frequency range between 5-6 Hz or greater [3]. The other four groups use one of these protocols: slow cortical potentials, mu and beta rhythms, cortical neuronal action potentials, and P300 evoked potentials [1]. Slow cortical potentials (SCPs) are the voltage shifts in the EEG that accrue over one to two frequency range. There are two types of SCPs, either negative shifts, which represent the decrease in the excitability, while positive shifts represent the increase in the excitability [3]. Mu and beta rhythms appear when a person is not engaged or busy with processing sensorimotor input or in producing motor outputs. The

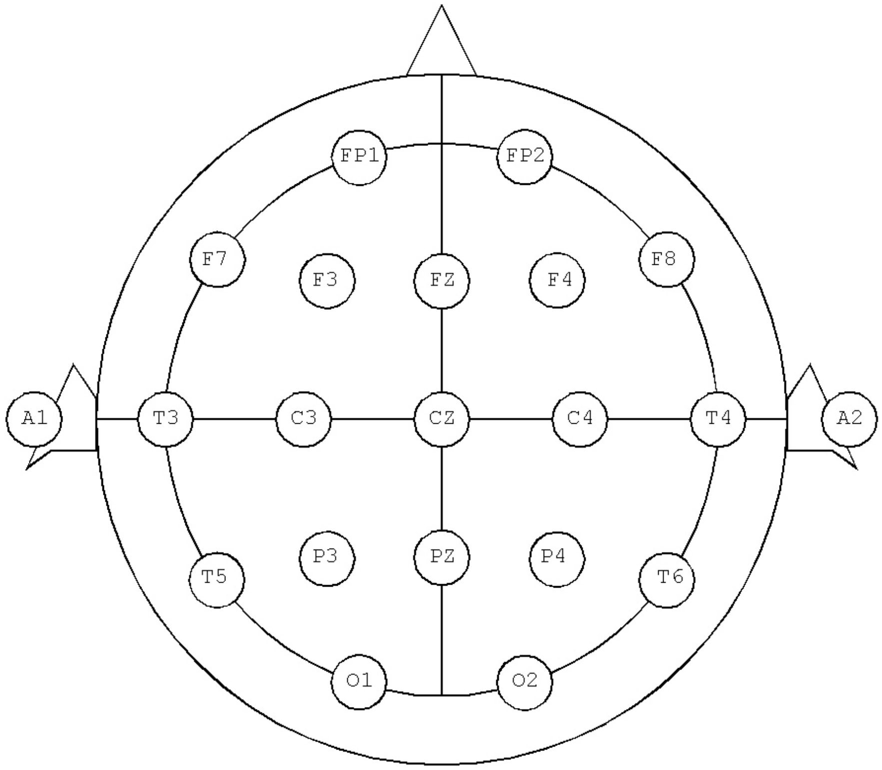


FIGURE 2.2. International 10-20 system for EEG [2].

frequency range for mu is between 8-13 Hz and beta is between 14-30 Hz. Cortical neuronal action potentials are firing rates of neurons in the motor cortex, which increase if movements are executed in the preferred direction of the neurons and decrease in the opposite case [3]. P300 is an event related potential (ERP) measure that measures the brain's response to a stimulus. P300 is detected in the oddball paradigm and a relatively large positive deflection in the voltage that starts about 300 milliseconds after the target stimulus is presented to the user (see Figure 2.3). To detect the presence of a P300 wave, a classification process is needed. Classification methods that have been used with BCI applications include linear methods such as LDA and Support Vector Machine (SVM); non-linear methods such as nonlinear Bayesian classifiers, Nearest Neighbor classifiers (K-nearest Neighbor), SVM as well as Neural Networks (NN); a combination of classifiers such as boosting, voting, and stacking [13].

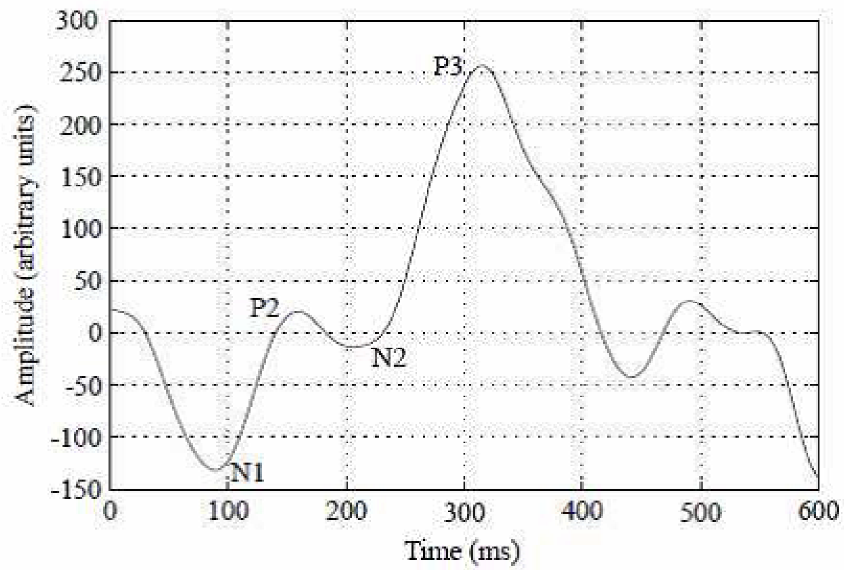


FIGURE 2.3. P300 wave indicated by P3. Other ERP, N1, P2, and N2 are also shown [3].

2.2. Subspaces and Singular Value Decomposition (SVD)

In order to compare between subspaces using the principal angles, the first step is the creation of the subspace based on the orthonormal condition. A Euclidean subspace of R^{n} is defined by a set of vectors with n real components, which are vectors from R^{n} [14]. The subspace (S) must have the following properties to be considered as a subspace:

- The zero vector is an element of the subspace S.
- S is closed under addition; this means that if x_1 and x_2 are vectors of the subspace S, then the result of the addition of these two vectors is also an element of the subspace S.
- S is closed under scalar multiplication. For example, if x is a vector in the subspace S then the result of any scalar multiplication, cx, is an element in the subspace S.

The four fundamental subspaces of a matrix ($A_{m \times n}$) are the column space (the span of the matrix columns), Null space of the matrix transpose (A^T), row space (the span of the matrix rows), and the Null space of the matrix(A) [14]. In Figure 2.4, all linear combinations of the columns of A (A_X) is the column space, C(A), while all linear combinations of the rows of A (A^Ty) is the row space, C(A^T). The orthogonal complement of a subspace S in R^n is the set of all the vectors of R^n that are orthogonal to every vector in the subspace S. The column space C(A) and the Null space of the matrix transpose N(A^T) (the set of all vectors y that satisfy $A^T y = 0$) are orthogonal, while the row space C(A^T) and the Null space of the matrix N(A) (the set of all vectors x that satisfy $Ax = 0$) are orthogonal. Because of this orthogonality, the equation $Ax = 0$ is defined, meaning each vector x in the null space is orthogonal to each row in the C(A^T). On the other hand, with the column space case $A^T y = 0$, which means each y vector in the null space of A^T is orthogonal to each column in the $C(A)$. The dimensionality of the row space is the row rank r, which is the maximum number of the linearly independent non-zero row vectors of the matrix A; and the dimensionality of the null space is determined by the number of columns minus the row rank. The same concept applies with the column space case; the rank is defined by the maximum number of linearly independent non-zero column vectors and the rank of the Null space of A^T is the number of the rows minus the column rank. Two vectors are orthonormal

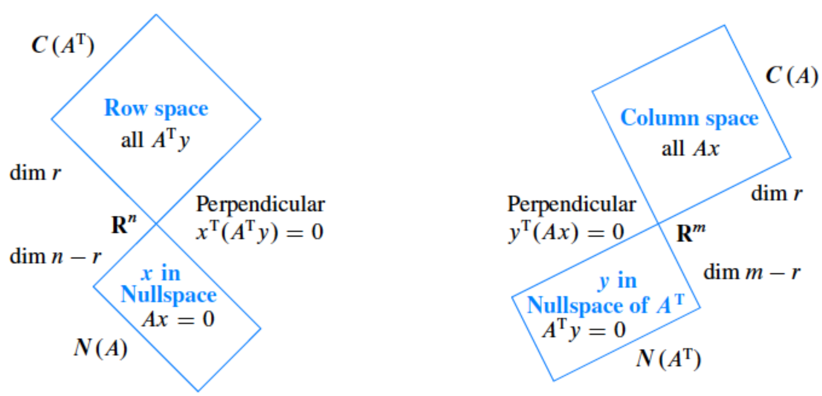


FIGURE 2.4. Dimensions and orthogonality for any m by n matrix A of rank r [4].

if they are orthogonal to each other and if they are unit length vectors. One of the methods that can be used to transform vectors to be orthonormal and get the orthonormal bases are the QR decomposition or factorization. A QR factorization of the matrix A is a product A = QR, such that Q is an orthonormal matrix, which are the first n columns of Q form the orthonormal bases of A and R is an upper triangular matrix. SVD is a factorization of the complex or real matrix A and is formulated as follows [14]:

$$A = U\Sigma V^T \tag{1}$$

$$AV = A\begin{bmatrix} v_1..v_r..v_n & \end{bmatrix} = \begin{bmatrix} u_1..u_r..u_m & \end{bmatrix}\begin{bmatrix} \sigma_1 & & \\ & & \\ & & \sigma_r \\ & & \end{bmatrix} = U\Sigma. \tag{2}$$

From the equation 2 we can see that the $U_{m\times m}$ and $V_{n\times n}$ are unit vectors. $\Sigma_{m\times n}$ is a diagonal matrix in which the values are non-negative real numbers. The diagonal values of Σ are the singular values, while U and V are the left and the right singular vectors,which are orthonormal eigenvectors of the matrix A [4].

2.3. Principal Angles

The concept of principal angles between subspaces was first presented by Jordan in 1875 [15]. The principal angle or canonical principal angle gives information about the relative position of two subspaces in a Euclidean space. If we have two subspaces, F and G, then the set of principal angles between these two subspaces can be defined as follows [14]. Let p be the dimension of subspace F and q be the dimension of subspace G. If we assume that $p \geq q \geq 1$, then the principal angles θ_1, θ_2, .. $\theta_q \in [0, \pi/2]$ between the subspaces F and G are defined recursively for *k=1,.,q* as

$$\cos(\theta_k) = u_k^T v_k \tag{3}$$

where

$$u_k, v_k = \underset{u \in F, v \in G}{\arg\max}\, u^T v, \tag{4}$$

subject to

$$\|u\| = \|v\| = 1, u^T u_i = 0, v^T v_i = 0, i = 1, ..., k-1 \tag{5}$$

The vectors u_1,...,u_q and v_1,...,v_q are called principal vectors. The first principal angle, θ_1, corresponds to the principal vectors (u_1, v_1). After that, the second principal angle will be found by searching in the subspaces for the vectors that are orthogonal to the first principal vectors; in other words, recursively always searching in the subspaces to find vectors that are orthogonal to the principal vectors that have already been found is necessary.

SVD can be used to calculate the principal angles by computing the cosine of the singular values[16]. Let Q_F and Q_G be orthonormal bases of F and G, respectively. Then

$$YSZ^T = Q_F^T Q_G \tag{6}$$

where S is a diagonal matrix with values along the diagonal of σ_1, σ_2, .. σ_q with 1$\geq$ σ_1 $\geq$ $\sigma_2$$\geq$, ...$\geq$ 0. The principal angles are

$$\theta_k = \arccos(\sigma_k), k = 1, ..., q \tag{7}$$

Knyazev [17] presents an algorithm, summarized in Figure 2.5, that was used in this study for determining principal angles in a more robust way when angles are very small or very large. Chang applied this algorithm in his study of image sets [5].

```
procedure PRINCIPAL ANGLES(F, G)                    ▷ F (n-by-p) and G (n-by-q)
    [Qf, Rf] = qr(F, 0);
    [Qg, Rg] = qr(G, 0);
    C = svd((Qf^T)Qg, 0);
    rkF = rank(Qf);
    rkG = rank(Qg);
    if rkF >= rkG then
        X = Qg − Qf(Qf^T Qg);
    else
        X = Qf − Qg(Qg^T Qf);
    end if
    S = svd(X, 0);
    S = sort(S);
    for i = 1 : min(rkF, rkG) do
        if ((C(i))^2 < 0.5 then
            angles(i) = acos(C(i));
        else if ((S(i))^2 <= 0.5 then
            angles(i) = asin(S(i));
        end if
    end for
end procedure
```

FIGURE 2.5. Small and large principal angles algorithm [5]

2.4. Principal Angles for Image Processing

Dealing with video streams and image sets as subspaces and comparing between these subspaces in order to know the similarities and differences is challenging and is a focus of many researchers for recognizing human faces and activities. There are many approaches that were implemented for learning over sets of vectors on the image set-based recognition. These approaches can be categorized into two groups: statistical and principal-angle based methods [18, 19]. Statistical based methods such as Kullback–Leibler Divergence (KLD) could be used after estimating the face appearance by multivariate Gaussian distribution for the linear case. KLD with the Gaussian Mixture Model (GMM) [5] or Resistor Average Distance (RAD) with the Kernel PCA [20] can be used for the non-linear case. These approaches rely on the assumption that images are independently and identically drawn samples from a probability density function. For solving the problem of set matching using these statistical methods, comparisons of the probability distributions of the probe set (images under different poses and illuminations for an unknown subject) and the gallery sets (a great number of images under different poses and illuminations for each known subject) are performed. The main drawbacks of these methods are that they need to solve difficult parameter estimation problems and they will fail if there is no good statistical relationship between the training sets and testing sets (example: there are variations in the expression and illumination). On the other hand, there are many approaches for the principal angle-based methods that calculate the principal angles between subspaces, such as Mutual Subspace Method (MSM) that takes the first principal angles as a similarity measurement between two linear subspaces. The drawback of this method is that the first principal angles that are taken are corresponding to the most similar modes of variations of the two subspaces, yet at the same time it might be caused by external factors such as extreme changes in the illumination conditions [21, 22]. In the Constrained Mutual Subspace Method (CMSM), transformations are applied to the vectors to maximize the separation between vectors from different classes by projecting the data with the generalized difference subspace. In this method, nothing is applied and considered for the vectors within class, which helps the similarity case. In addition, the classification accuracy is affected by the generalized different subspace dimensionality. Kernel Principal Angle (KPA) is also one of the principal angle approaches that finds and calculates the principal angles between nonlinear subspaces after mapping data from the original space to a nonlinear feature space. The limitation of this work is that the evaluation was performed on a small size dataset, so judgment cannot be taken on these results since it might not accurately work with a large size dataset. In addition, finding the optimal kernel function is a difficult process.

Kim, et al., [23] use the same idea of transforming the data as in CMSM, but this new method considers both cases within and between classes. They proposed one of the principal angle approaches for comparing image sets in order to recognize objects and faces under different illuminations and camera poses. Researchers in this approach create a linear discriminant function to transform data to maximize the canonical correlation (small value for principal angles) within the class sets and minimize the canonical correlation (large value for principal angles) between class sets. In this method, authors presented a highly time efficient algorithm for matching between subspaces. In their method, there are no features that need to be selected and full dimensionality can be used. They used the discriminative function that makes their method more robust. However, this method works with the linear subspaces, which might not work well with complex cases that cannot separate the subspaces linearly. Kim, et al., [22] solve this problem by presenting Boosted Manifold Principal Angles (BoMPA) method that combines both non-linearity patterns and discrimination concepts using the AdaBoost algorithm. A drawback of this method is that the authors used AdaBoost algorithm, which is sensitive to noise and outliers [24, 25].

Chang and Pacheco [26] use the concept of finding the similarities and differences between the subspaces using the distances between the Grassmann manifolds by using the principal angles between them to classify the handwritten digits. These authors present two different algorithms for comparing and classifying the testing data either one to one, which are vector to subspace, or many to many, which are subspace to subspace. Before they measure the principal angles, they apply four different transformations (rotation, scaling, and horizontal and vertical translations) on each vector in the training data and each vector in the testing data to mimic variations in human handwriting. For comparing between testing vector and training subspaces, the authors directly compare the calculated principal angles. On the other hand, in order to compare between testing and training subspaces, the comparison was performed between the distances, which are based on the principal angles between these subspaces. Many algorithms and methods have been implemented to calculate the principal angles between subspaces, usually followed by the use of nearest neighbor algorithms based on principal angles to classify the image sets such as this study. The comparison with other classification methods is needed to evaluate the presented method better.

2.5. Principal Angles for EEG Analysis

Recently, some works have been presented that use principal angles between subspaces for EEG analysis to determine the similarities and differences between them. Anderson and Kirby [27] recorded EEG signals from six channels while the subject was doing two different mental tasks, which were a multiplication task (non-trivial multiplication) and an imaginary letter-writing task. In this study, EEG data was represented as a subspace using SVD. Different variations were applied such as using different numbers of electrodes, different lag values (overlapping between windows), and different modes, which means different numbers of eigenvectors are created after using the SVD algorithm. Moreover, different protocols were proposed such as recording the data on the same or different days for training and testing data in order to evaluate how these variations affected the classification performance. Data was transformed using two different methods Karhunen- Loe‘ve (KL) and Maximum Noise Fraction (MNF) in order to compare the results and observe which method provides the most accurate results. After the data had been transformed, classification was done using Quadratic Discriminant Analysis (QDA). This study did not consider principal angles to show the similarities and differences between target and nontarget subspaces or between different task subspaces. In addition, the authors show that classification results vary from one day to another for the same person. However, this study tested on one subject and as mentioned before EEG patterns change from one person to another.

Samek, et al., [28] show that nonstationary subspaces are somehow similar between different subjects, while the discriminant subspaces, which were spanned by the Common Spatial Patterns (CSP) filters, are quite different between subjects. With this result the authors estimated the changes between the training and testing sessions for one user using the information from another user. The information that can be transferred between users is nonstationary since they are similar between subjects based on the principal angle results. On the other hand, the discriminant information cannot be transferred between subjects. Authors tested their approach on two datasets of EEG recordings from subjects while performing motor imagery and used LDA as a classifier. The first dataset is the motor imagery of moving two limbs, specifically the left hand and foot. Subjects responded to either the stimulus that was presented visually as an arrow on the center of the screen or auditorily as a voice announces the task that should be performed. The second dataset was Dataset IVa from BCI Competition III [29] of five subjects performing right hand and foot moving imagery. The authors reported that by estimating the nonstationary information and removing this information from the data, the classification accuracy was improved. However, all three transfer methods that are presented in this study are critical to the choosing of the regularization parameter especially if the subject similarity is low. Referring to this problem, the classification performance will be affected. In addition, the stationary subspace CSP (ssCSP) is limited to the maximum dimensionality of the nonstationary subspace that are removed from the data, which affect the amount of information that can be transferred.

Liy, et al., [30] worked with five different EEG data sets, four related to different diseases and one normal. Kernel principal angles between the normal data and the four diseases were found to show differences between these two data sets. In addition, kernel principal angles were calculated between the testing data and all five subspaces to recognize if the data was more closely related to the normal signals, the testing signals, or to one of the four diseases. Kernel principal angle methods produce principal angles between nonlinear subspaces after mapping data from the original space to a nonlinear feature space. Authors in this study derived a sparse kernel principal angle (SKPA) method to calculate the principal angles in the feature space. The idea of the SKPA is to first map the input space to Hilbert feature space using a nonlinear mapping method after that compute the basis of the feature space. For each subpace, basis selection method was applied using equation 8. The basis ϕ is selected only if the resulted value from the equation 8 is less than the sparse sensitive parameter (ϵ), which is a very small positive number that is closed to zero. A limitation of this approach is the difficulty in finding the optimal kernel function [31]. In addition, the results are not clear enough and more details are needed for both the experiments and results descriptions.

$$\min(\varphi(a_k) - \sum_{\varphi(a_i)\in A_d} \lambda\varphi(a_i))^T(\varphi(a_k) - \sum_{\varphi(a_i)\in A_d} \lambda\varphi(a_i)). \tag{8}$$

Chuang, et al.,[32] use the principal angles for the authentication purpose and distinguish the EEG signals among different users. The authors present a study that recorded the EEG signals using the Bluetooth EEG headset. Fifteen subjects were asked to complete seven mental tasks such as breathing, finger movement simulating, sport task imagining, singing, listening to audio, counting, and choosing special password. They calculated two parameters based on the principal angles, which are self-similarity that find the similarity of the recorded signals within the subject and cross-similarity that find the similarity between different subjects in all tasks. The authors found that the value of the self-similarity is higher than the cross-similarity value, which helps in the authentication process. Furthermore, they found that the variation of the similarity between subjects is higher than the variation between tasks. However, in the pass through task where subjects are asked to choose their own password and focus on their password for 10 seconds when there were taught, more verification steps are needed to eliminate the problem of the attacker and misclassification that could lead to another person to be identified. In addition, it will be helpful and useful if the comparison with other classification methods is included better understand how the performance of the presented study competes against other methods.

2.6. Other approaches for P300 classification

Different research had been studied on the P300 BCI applications to detect the P300 signal using different classification methods either linear or nonlinear such as, LDA, linear support vector machine (SVM), Gaussian kernel support vector machine, and NN [33]. Bakhshi and Ahmadifard[34] are also compared between the linear and nonlinear classification methods. Authors tested a six choice P300 paradigm on five impaired and four unimpaired subjects. Authors used Fisher LDA (FLDA), Bayesian LDA (BLDA), and NN to classify the P300 signals. Bandpass filter was implemented to filter the signals and the cut-off frequencies were set to 1.0 and 12.0 Hz. In addition, the EEG data was down sampled from 2048 Hz to 32 Hz through selecting the 64th sample from the filtered data. Different number of channels were tested such as, 4,8,16, and 32. Comparing between the mean of the classification accuracy for different electrode configurations, NN is lower than the two other methods, while BLDA is higher than the FLDA with little difference. In addition, increasing the electrode number to 32 did not add any benefit to increase the classification accuracy. The highest accuracy for BLDA was 95.7% using 16 channels, 94.5% using 16 channels for FLDA, and 89.1% for NN using 4 electrodes.

Ou Li, et al., [12] used the combination of the median filtering as a preprocessing method to remove the noise from the data and BLDA to classify the signals. The median filter is a nonlinear filtering method, which kept the signal that are greater or equal to $K + 1$ if the window length is $2K+1$ (odd case) or $(K+K+1)/2$ if the window length is $2K$ (even case), otherwise the signal will be removed. For example, if the data are $\{2, 1, 4, 1.5, 2.5\}$ for the filtering window $= 5$, then the median filter is 2 that is related to $(K + 1)$. Authors tested their presented method on the P300 speller paradigm in dataset II of BCI Competition III [29]. The target letters were D, E, O, R, and U. The median filter method was applied, where the filter window length is 50. K- Fold cross validation, for k= 2, 3, and 4, was used to divide the data into different training and testing setup. After using BLDA method to classify the testing data, 100% as a classification accuracy was reached in some case and 90% average classification accuracy was obtained. Authors also showed that their results are better than combining Bandpass filter and BLDA.

Hutagalung and Munandar worked on an online data using nonlinear principal component analysis (NPCA) as a feature extraction method and backpropagation neural networks as a classification method without any of the down-sampling or averaging preprocessing steps [35]. The data was recorded from seven healthy subjects while they were responding to seven different choices such as: forward, turn right, turn left, backward, backward right, backward left, and stop. Eight electrodes were used to record the EEG signals at Fz, Cz, Pz, Oz, P7, P3, P4, P8 with 256 Hz as a sample rate. Data are first preprocessed using the banspass filter with cut off frequencies between 1 and 12 Hz. After that NPCA is applied to extract the features. There are four main steps in NPCA. Pre-separation step that uses the multi stage PCA in the first layer in order to reduce the artifacts and noise from the signal. Whitening step on the second layer uses the PCA to remove the redundancy from the observed data.

Separation step that is applied on the third layer using the NPCA to separate the whitened signals. The last step is applied on the fourth and fifth layers the independent component basis vector that are coming from the mixing matrix is estimated. Authors reported that using their proposed method the classification accuracy was greater than 80% to detect and classify the P300 signals.

Farooq and Kidmose proposed the use of the Random forest (RF) as a classification method [36]. The RF training model is started with linking each of the P300 and nonP300 signals to the correct label. After that, using the bootstrap method with replacement to select the independent randomly sets from the whole training data. Later, the tree is constructed based on the selected set. GINI criterion is implemented to choose the optimal way to generate the child nodes. The decision is finally taken based on the majority voting. One of the datasets that the authors tested the proposed classification method on is the BCI competition II for two subjects. All of the 64 channels are used and bandpass filter between 0.1 and 10 Hz and averaging step was applied as a preprocessing steps. The authors tested the proposed method with other P300 BCI known classification methods such as: SVM, the step-wise linear discriminant analysis (SWLDA), the multiple convolutional neural networks (MCNN), and the ensemble support vector machine (ESVM). Both the classification accuracy and the information transfer rate (ITR) in bits/minutes are measured. In most cases the RFE gives the best classification accuracy and high ITR comparing with the other classification methods. The average classification accuracy for RF was 97.85%.

All of the classification accuracies reported above are achieved by classifying multiple trials. If only a single trial is used, the classification accuracy is 60% using the approach that had been implemented by Hutagalung and Munandar [35] and the classification accuracy is equal to or less than 30% for the other two approaches [12] [36]. Nearest training subspace can be found using the smaller value of the calculated principal angles. The balanced classification accuracy using a single or small number of trials in each subspace can be better or close to the accuracies that were reached by these classification methods that used more trials. This concept makes the principal angle approaches more suitable for online BCI applications.

CHAPTER 3

Methods

This chapter summarizes the experiment protocols and methods for collecting and preprocessing EEG, calculating principal angles, and using the principal angles to remove outliers and to detect the presence of P300 waves.

3.1. Experimental Data

The experiments described here involve data from four subjects, two with impaired motor functions and two without. The recording of EEG for the subjects with impairment was performed in the subjects' homes; recording for the other subjects was performed in a university lab. All subjects signed an informed consent form approved by the IRB of Colorado State University. Table 3.1 shows the subjects' demographic information.

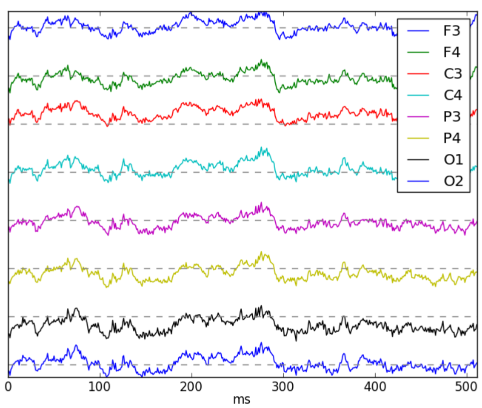
The g.GAMMAsys and g.MOBILab+ system by G.tec were used to record eight-channel EEG from sites F3, F4, C3, C4, P3, P4, O1, and O2, referenced to the left earlobe, with a 256 Hz sampling rate. The work reported here only used data from P4 since, of the eight channels, it is often considered to be most relevant for P300 detection [37]. Custom software, written in Python, was used to present visual stimuli and acquire the EEG signals [38].

Subjects sat about three feet from a computer display. To elicit P300 waves, subjects were asked to count the number of times a particular letter was flashed on the screen. Each letter appeared for 100 ms, followed by a blank screen for 750 ms. A total of 80 letters

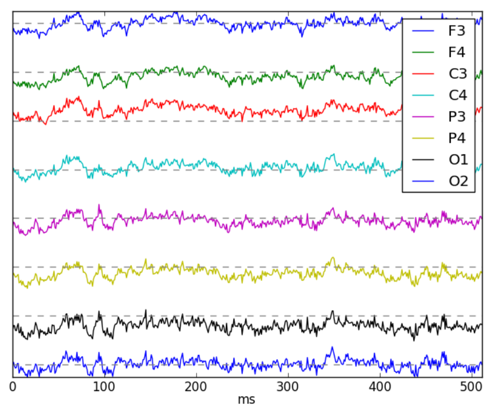
Table 3.1. The subjects demographic information.

Subject No.(No. in the downloaded data)	Gender	Impaired	Functional disability
Unimpaired Subject 1	M	No	–
Unimpaired Subject 2	F	No	–
Impaired Subject 1	M	Yes	Spinal Cord injury C4
Impaired Subject 2	M	Yes	Multiple Sclerosis (MS)

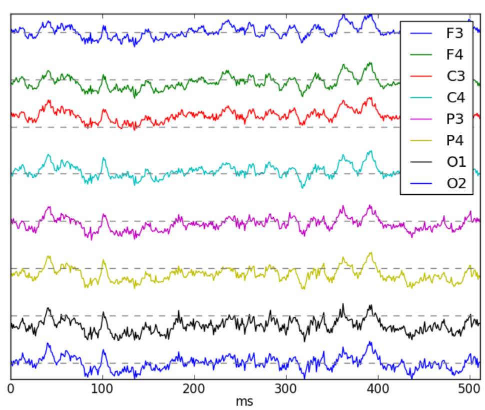
were flashed, with 20 being the target letter and 60 being other letters. The order of letter presentation was randomly chosen. This procedure was repeated three times with the target letter being b, then p, then d. These three letters were chosen as target letters because they have similar shapes with different rotations, thus minimizing differences in EEG due to perception. The objective is to detect the occurrence of a target letter, not to identify the particular target letter, b, p, or d. Figure 3.1 illustrates an example of the recorded EEG signals from an unimpaired subject using the eight channels for the three letters. The stimulus occured at times 0.



(A) Letter b



(B) Letter p



(C) Letter d

FIGURE 3.1. Sample of unimpaired EEG Signals.

Since the period between the stimuli (appearance of letters) is 850 ms and the sampling rate is 256 Hz, EEG was segmented into windows of 210 samples starting from the stimulus onset. Each 210 sample window of EEG will be called a *trial* in the remainder of this dissertation. For example, Figure 3.2 presents some of the P300 target trials and some others that are non P300 trials.

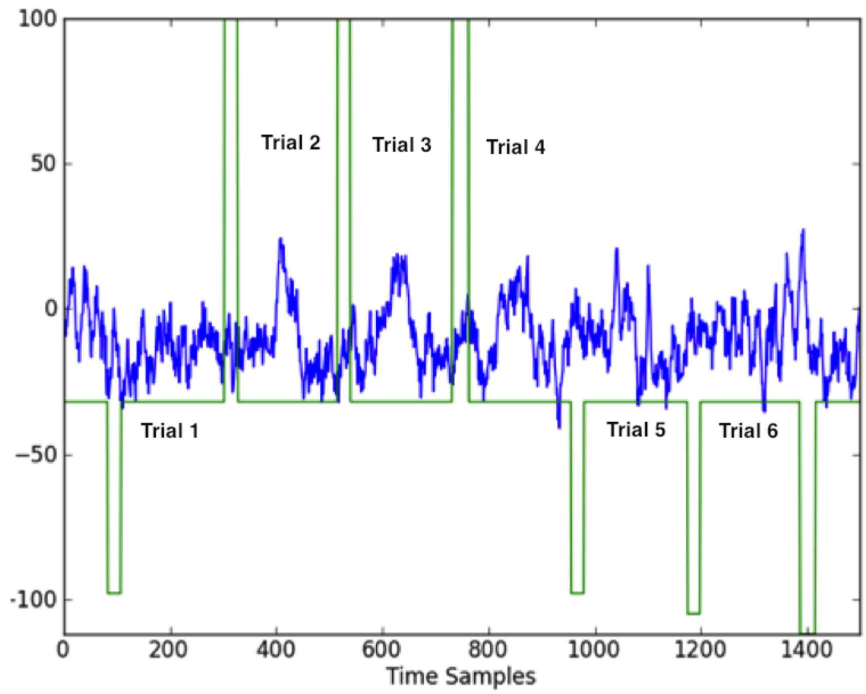
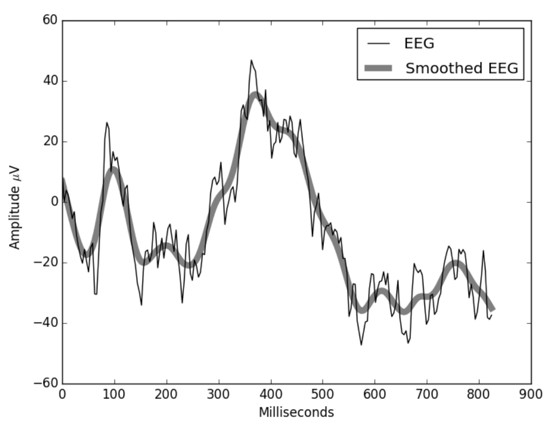


FIGURE 3.2. Trials 2, 3, 4 are target. Trials 1, 5, 6 are nontarget.

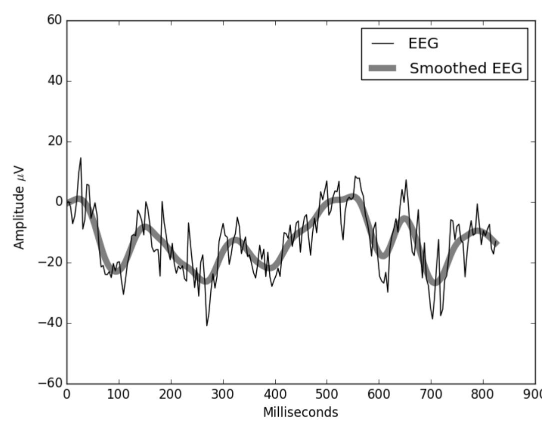
3.2. Preprocessing

Since the EEG trials are often noisy, the detection of the P300 wave is not easy. Before creating the subspaces and calculating the principal angles between these subspaces, smoothing in time is implemented on the EEG signals to improve the P300 data and help with analysis process. For BCI applications, averaging over multiple trials increases the time required for the user to make a selection, so is to be avoided. However, smoothing can be performed on individual trials. Here, data was smoothed using Stickel's [11] algorithm, which smoothed data using regularization. As shown in equation 9, $y(x_i)$ is a set of data, for $i = 1, 2, \ldots, N$, $y(x)$ is the function that described the data trend, $\hat{y}(x_i)$ is the smooth function that fits and approximates $y(x_i)$. This method approximates the data by minimizing a squared error formulated by the first part of the equation and a roughness measure defined by the second derivative of the approximation ($d = 2$) as displayed in the second part of the equation. The algorithm requires an empirically chosen regulation parameter λ. A value of 0.0001 was used in this study since it resulted in the best classification accuracy. This algorithm was applied to all trials, both those containing a P300 and those not containing a P300. Figure 3.3 illustrates the results of the smoothing algorithm on one of the subjects that was used in this dissertation [39].

$$Q(\hat{y}) = \Sigma_{i=1}^{N} |\hat{\mathrm{y}}(\mathrm{x_i}) - \mathrm{y}(\mathrm{x_i})|^2 \, \mathrm{d}x + \lambda \Sigma_{i=1}^{N} |\hat{\mathrm{y}}^{(\mathrm{d})}(\mathrm{x_i})|^2 \, \mathrm{d}x \tag{9}$$



(A) a P300 trial



(B) a non-P300 trial

FIGURE 3.3. Results of the smoothing algorithm.

The data first is divided into target and nontarget subspaces. For each letter and each subject, the target subspace consists of 20 trials and the nontarget contains 60 trials. As mentioned before, only one channel is used and one trial consists of 212 samples. Figure 3.4 shows a simple illustration of a target subspace with the three P300 trials and a nontarget subspace with three non P300 trials. As shown in this figure, the principal angle between these subspaces is a large value as shown in the figure since these two subspaces are different. U and V are the principal vectors that are related to this principal angle.

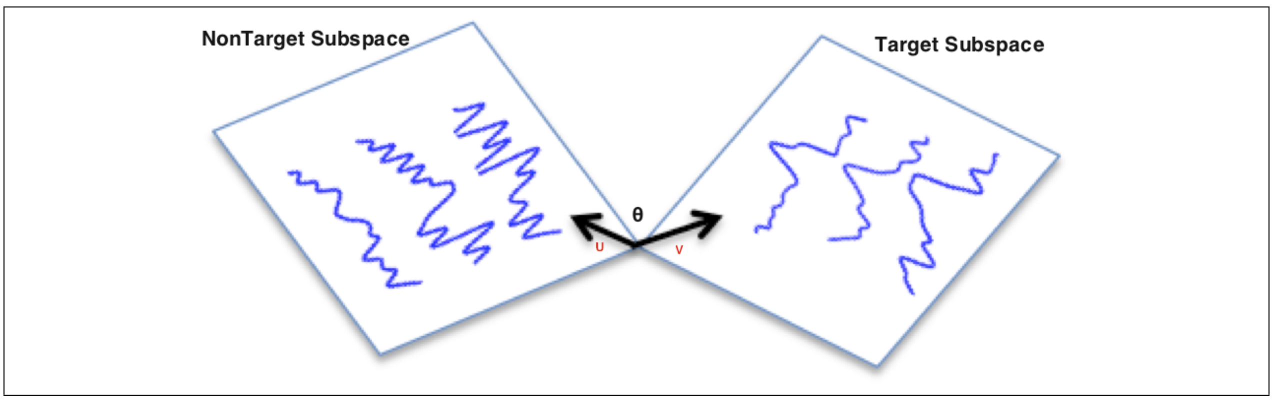


Figure 3.4. Target and NonTarget Subspaces Example

The following scenarios are presented in this dissertation.

(1) For each subject separately, all target trials for all three letters were combined to define the target subspace and all nontarget trials were also combined to define the nontarget subspace. For each of the three target letters, 20 target trials and 60 nontarget trials were collected, for a total of 60 target and 180 nontarget trials or 240 trials total. After that, the target and nontarget trials were partitioned into five disjoint subsets, resulting in 12 trials of target subsets and 36 trials of nontarget subsets. Subsets were collected into training and testing sets by combining the ith target and nontarget subset to form the testing set and the remaining subsets to

form the training set, for $i = 1, 2, \ldots, 5$. Thus the training sets contain 48 and 144 target and nontarget trials, respectively, while the testing sets contain 12 target and 36 nontarget trials.

(2) To test the ability of a classifier trained on two target letters to generate to an untrained target letter, the following section was used. In this scenario, letters p and b are the only two letters that form the training subspace and letter d forms the testing subspace. In this case, the 20 testing target and the 60 nontarget trials are divided into five disjoint trials to form 4 target and 12 nontarget trials in each turn, while the training subspace contains 40 target and 120 nontarget trials.

(3) One of the dissertation contributions is to study and evaluate the transformation of data between subjects, which means training the method on subjects that are different from the testing subject. Similar to the first scenario with the combination of all three letters, in the third scenario, the training was on three subjects and the testing was on the remaining subject. This means that the training target subspace is the combination of 180 target trials for the three subjects, and 540 nontarget trials. Similar to the testing trials that are related to one subject, the target subspace is 60 trials and the nontarget is 180 trials. The concept of partitioning the subspace to five disjoint set was also implemented in this scenario, which resulted in 12 target trials and 36 nontarget trials.

In all the above scenarios, each subspace was formed using different number of trials per subspace in order to have the fewest number of trials with a high classification accuracy that helps in the online applications. The process was repeated 16 times, each time the training and testing subspaces created using different numbers of trials such as 1, 2, 3, 4. Tables 3.2 and 3.3 describe the training and testing subspaces' sizes in each of these four cases.

TABLE 3.2. Number of Training Subspaces. The Number of Trials is The Trials That are Needed for Creating The Subspaces.

	No. of Trials =1		No. of Trials =2		No. of Trials =3		No. of Trials =4	
Scenario	Target	NonTarget	Target	NonTarget	Target	NonTarget	Target	NonTarget
1	48	144	24	72	16	48	12	36
2	40	120	20	60	13	40	10	30
3	180	540	90	270	60	180	45	135

TABLE 3.3. Number of Testing Subspaces. The Number of Trials is The Trials That are Needed for Creating The Subspaces.

	No. of Trials =1		No. of Trials =2		No. of Trials =3		No. of Trials =4	
Scenario	Target	NonTarget	Target	NonTarget	Target	NonTarget	Target	NonTarget
1	12	36	6	18	4	12	3	9
2	4	12	2	6	1	4	1	3
3	12	36	6	18	4	12	3	9

The number of the principal angles that are created is based on the smallest size of the training and testing subspaces. For example, in the case that both training and testing subspaces consist of a pair of trials, two principal angles were created that were used to reject the outliers and detect the P300 wave, while only one principal angle can be calculated between subspaces that when testing trial size is 1 and the training trial size is 2. When partitioning into set of three trials, the number of trials is not evenly divisible by three. Based on this in some situations in the second scenario, a remaining trial was removed to form the subspaces that contain three trials only.

After defining both target and nontarget training and testing subspaces, the principal angles between these subspaces are calculated using Algorithm 2.5 that was defined in Chapter 2. Different measures based on the principal angles can be defined to find the similarity between a subspace and a set of other subspaces. The number of the created measures is also based on the smallest subspace size. For example, to determine the similarity between testing subspace A that consists of 3 trials and training subspaces B_1, B_2,..., B_n that consist a pairs of trials, four similarity measures were defined:

- the minimum of the first principal angles,
- the minimum of the second principal angles,
- the mean of the first principal angles, and
- the mean of the second principal angles.

Figure 3.5 shows a simple example of a toy data. This data consists of two subspaces A and B as positive and negative subspaces. Each subspace contains five vectors. The two subspaces are described below.

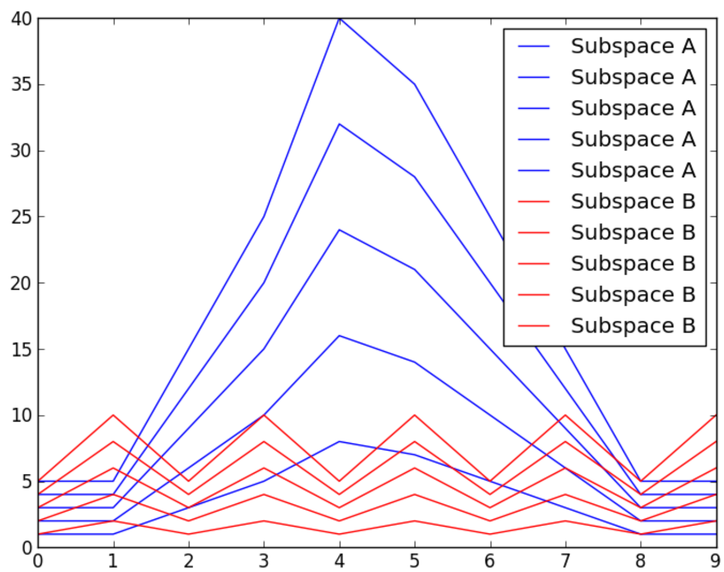
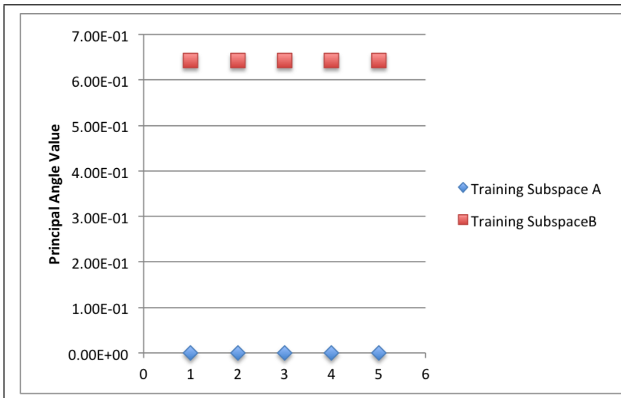
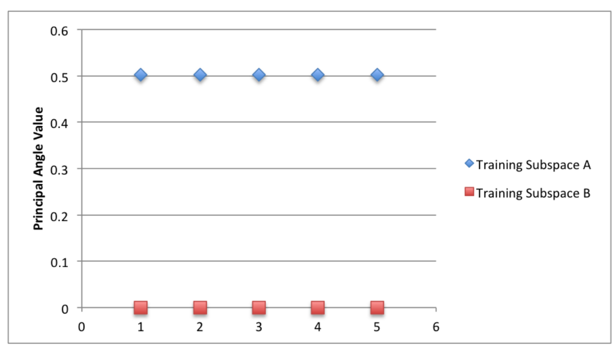


FIGURE 3.5. The toy data. Subspaces A are P300 trials and Subspaces B are non P300 trials

$$(10)\quad A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 3 & 6 & 9 & 12 & 15 \\ 5 & 10 & 15 & 20 & 25 \\ 8 & 16 & 24 & 32 & 40 \\ 7 & 14 & 21 & 28 & 35 \\ 5 & 10 & 15 & 20 & 25 \\ 3 & 6 & 9 & 12 & 15 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix} B = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 6 & 8 & 10 \\ 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 6 & 8 & 10 \\ 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 6 & 8 & 10 \\ 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 6 & 8 & 10 \\ 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 6 & 8 & 10 \end{bmatrix}$$



(A) The testing trial is A



(B) The testing trial is B

FIGURE 3.6. Prinicpal angle values using a toy data

Recursively one vector is excluded from the subspace A and the same situation goes for subspace B to create the testing subspace. The remaining four vectors from each subspace are used to create the training subspaces. Principal angles are calculated between the testing and training subspaces. The principal angle value in the similar case should be smaller than the different case. As shown in Figures 3.6a and 3.6b, if the testing vector is related to the subspace A, the principal angle values when the comparison is done with the training subspace A are smaller than training subspace B. The same results are shown if the testing vector is related to the subspace B. Based on this, the classification rate is 100% in both cases.

3.4. Outliers Detection and Rejection

One of the points that was tested in this dissertation is the effect of the outliers on the proposed method. Some EEG trials contained artifacts, such as large deviations because of eye blinks or neck muscle twitches. Some trials can be defined as outliers in the distribution of the normal trials; removing these trials from the training subspace might help to increase the classification accuracy. The outliers in this study were detected based on the principal angles and the created similarity matrices between either target or nontarget training trials.

To find the outliers from the training subspaces either target or nontarget, principal angles should be first calculated within the same subspace. This means finding the principal angles between all the sets of the target subspace and all the sets of the nontarget subspace. For example, if the target training subspace consists of 20 trials then the dimensionality of the created principal angles comparison matric within this subspace for the two trials case is $20 \times 19 \times 2$. Furthermore, if the nontarget trials are 60 then the dimensionality of the principal angle matric is $60 \times 59 \times 2$. After that, the four measures above were defined and the outliers were detected based on these four measures. If the principal angle value is more or less than one standard deviation away from the mean of the measure among the training set subspaces, then this subset is removed. Equation 11 illustrates this concept and the results of this step will be discussed in the next chapter. PA_i is the minimum or the mean of the principal angles which are defined by the similarity measures of the four different cases that previously explained. μ_i and σ_i are the mean and standard deviation of PA_i.

$$\mu_i - \sigma_i \leq PA_i < \mu_i + \sigma_i \tag{11}$$

3.5. Classification and Classification Accuracies

After smoothing the signals and removing the outliers, the classification process is performed. In each scenario, the principal angles between all of training target and nontarget subspaces and each testing subspace are calculated. Then the similarity measures defined above are created for both the target and nontarget cases. Based on these measures the test subspace is classified as a target if the value of the similarity measures between the test subspace and the train target subspace is smaller than the case when comparing the test subspace with the train nontarget subspace, otherwise it is misclassified as a nontarget subspace.

Different methods were defined to classify and label the testing subspace. For example, in the case that the subspaces are created using pairs of trials, the classification is performed based on the minimum of the first principal angles, the minimum of the second principal angles, the mean of the first principal angles, and the mean of the second principal angles. After trying all these four measures, it was determined that the mean of the first principal angles resulted is the best accuracy. Results of this classification is the one that are shown in detail in the next chapter.

Straube and Krell [40] showed that the accuracy performance should be calculated in a way that is not sensitive to the size differences of the classes. Since the number of nontarget trials is three times the number of target trials, the balanced accuracy (BA) is preferred to the normal accuracy [ACC] equation. Results for both BA and ACC are presented in the next chapter.

$$ACC = \frac{TP + TN}{TP + FP + FN + TN} \tag{12}$$

$$BA = (TPR + TNR)/2 \tag{13}$$

$$TPR(TruePositiveRate) = \frac{TP}{TP + FN} \tag{14}$$

$$TNR(TrueNegativeRate) = \frac{TN}{TN + FP} \tag{15}$$

The true positive (TP) is the number of positive cases (target) that are correctly classified as positive (target). The true negative (TN) is the number of negative cases (nontarget) that are correctly classified as negative (nontarget). The false negative (FN) is the number of positive cases (target) that are incorrectly classified as negative (nontarget). The false positive (FP) is the number of negative cases (nontarget) that are incorrectly classified as positive (target).

CHAPTER 4

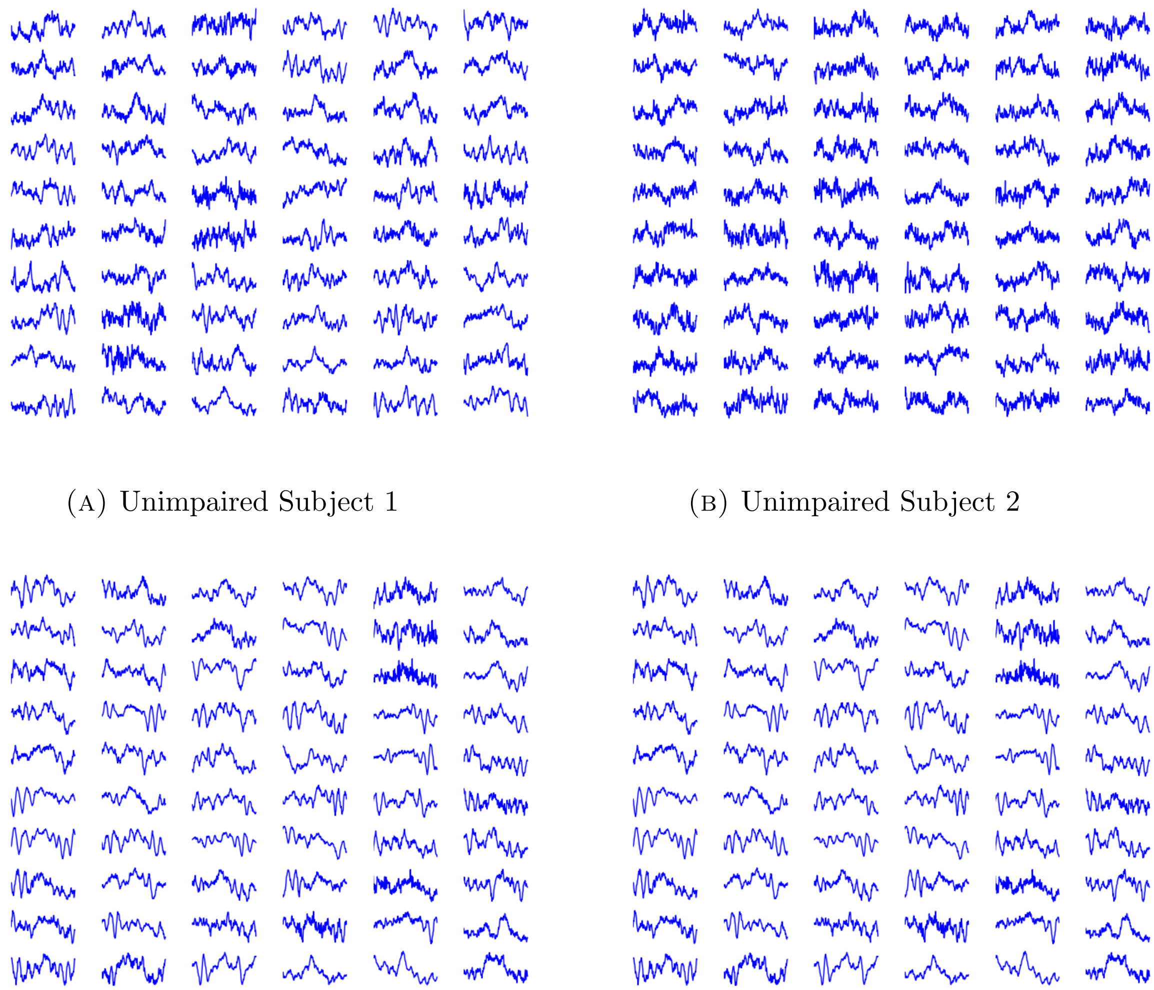
Results and Discussion

In this chapter, the results of the proposed method that were discussed and outlined in Chapter 3 are presented. In addition, the discussion and explanation of these results will be mentioned. In this chapter, different focuses are studied: first, comparing the presented preprocessing smoothing function with the FIR Bandpass filter method, second, evaluating the classification accuracy that depends on the principal angles and principal angles outliers removal, third, studying the effecting of increasing the trial sizes to create both training and testing subspaces. In this point, all scenarios will be examined such as, evaluating the case that both training and testing subspaces consist of the three target letters, studying the success of the proposed method when the testing scenario become harder such as training on two different letters and testing on the third letter, and testing the transferring information concept between subjects such as training on three subjects and testing the presented method to classify trials related to the fourth subject. Finally, comparing the results of the proposed method with other classification method such as Linear Logistic Regression and other recording system such as Biosemi Active Two.

4.1. The comparison between the preprocessing methods

EEG signals are noisy as mentioned before and need to be smoothed before applying the proposed method to detect the outliers and classify the testing signals. In this section, two methods are compared, the bandpass FIR filter and the smoothing by regularization method that was described in Chapter 3.

Figure 4.1 illustrates the EEG signals for the 60 target trials that are related to all of the three target letters for all four subjects before applying any of the preprocessing methods.



(A) Unimpaired Subject 1

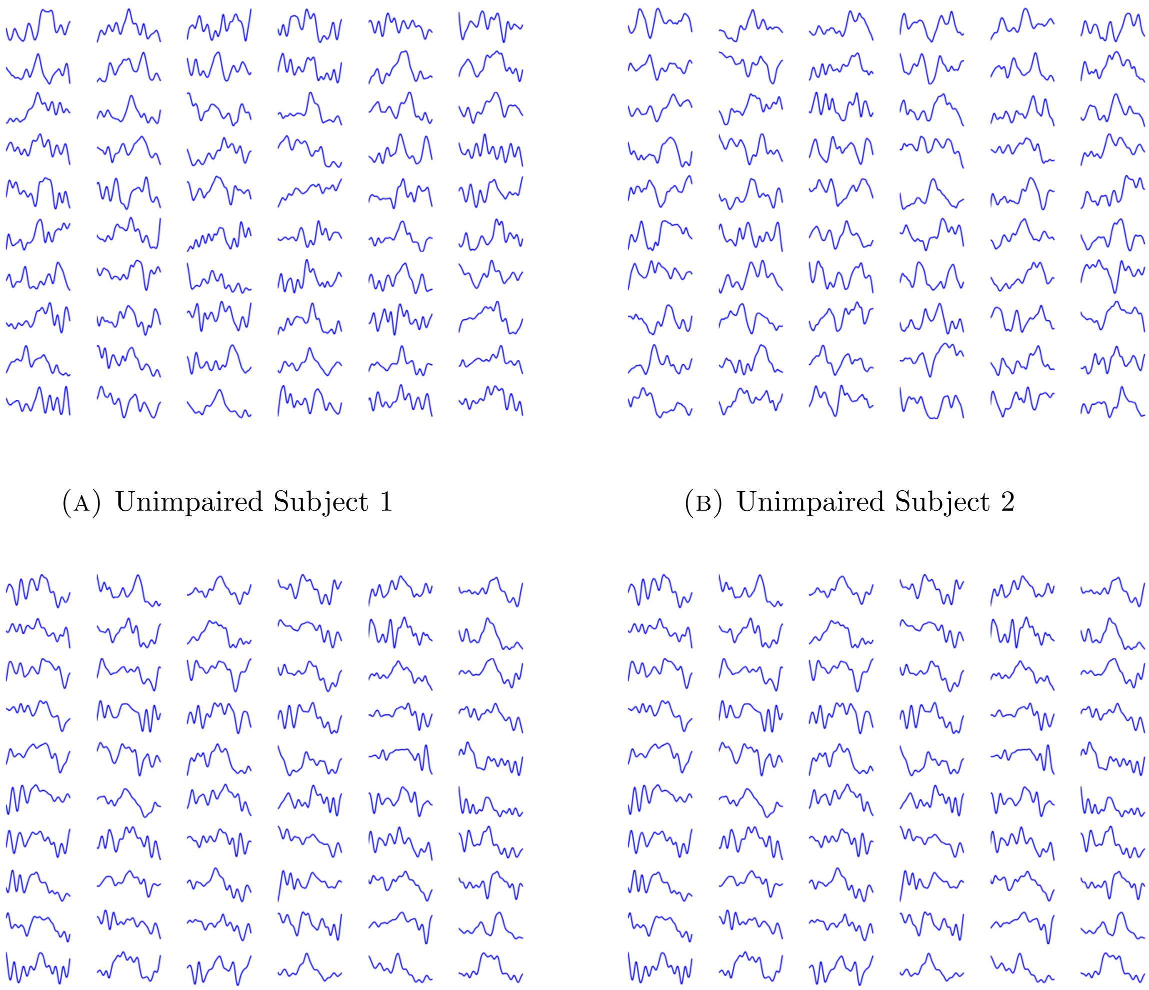
(B) Unimpaired Subject 2

(C) Impaired Subject 1

(D) Impaired Subject 2

FIGURE 4.1. Target EEG Signals

Figure 4.1c shows that the Impaired Subject 1's EEG signals appear to give a good P300 representation, while Figure 4.1d demonstrates that the Impaired Subject 2's signals seem to display a poor P300 representation. Figure 4.2 shows the same signals after applying the presented preprocessing method and Figures 4.3 and 4.4 show the same signals after applying a bandpass filter using different ranges. In Figure 4.3 the range of the bandpass filter was between 0.5 and 30.0 Hz and the bandpass filter in Figure 4.4 is between 0.5 and 10.0 Hz.

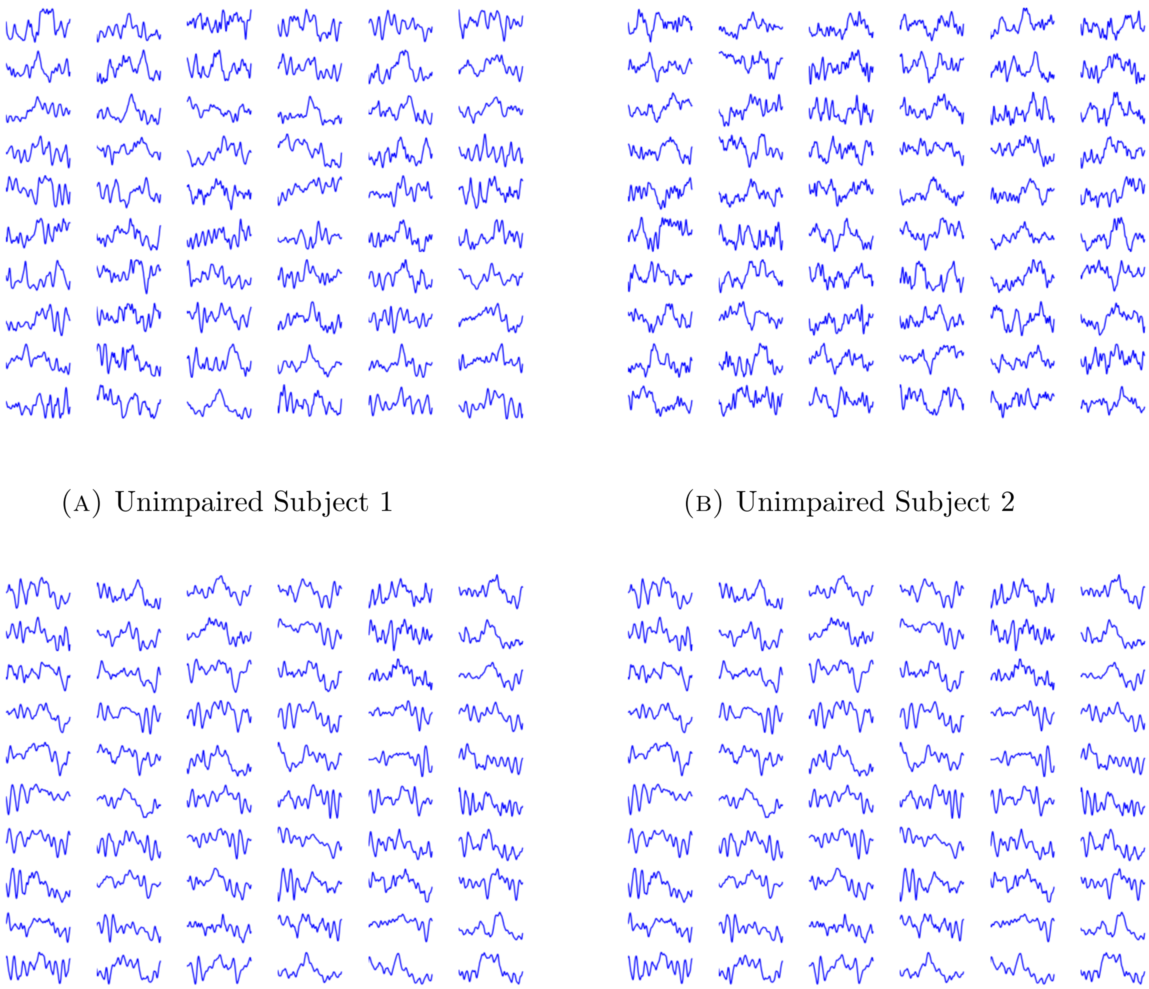


(A) Unimpaired Subject 1 (B) Unimpaired Subject 2

(C) Impaired Subject 1 (D) Impaired Subject 2

FIGURE 4.2. Target EEG Signals after applying smoothing by regularization method

In this section, most of the concentration is on classifying either the target or nontarget testing subspace that contains one trial only. The classification is done by comparing the testing subspace with different trial sizes of training subspaces such as 1, 2, 3, and 4. Results displayed in this section consider three scenarios that were mentioned in the previous chapter. Table 4.1 and Table 4.2 show the classification accuracies of the three scenarios for the impaired Subject 1 since this subject has a good P300 representation. Comparing these tables, no clear trends can be found when comparing the bandpass filter with range between



(A) Unimpaired Subject 1

(B) Unimpaired Subject 2

(C) Impaired Subject 1

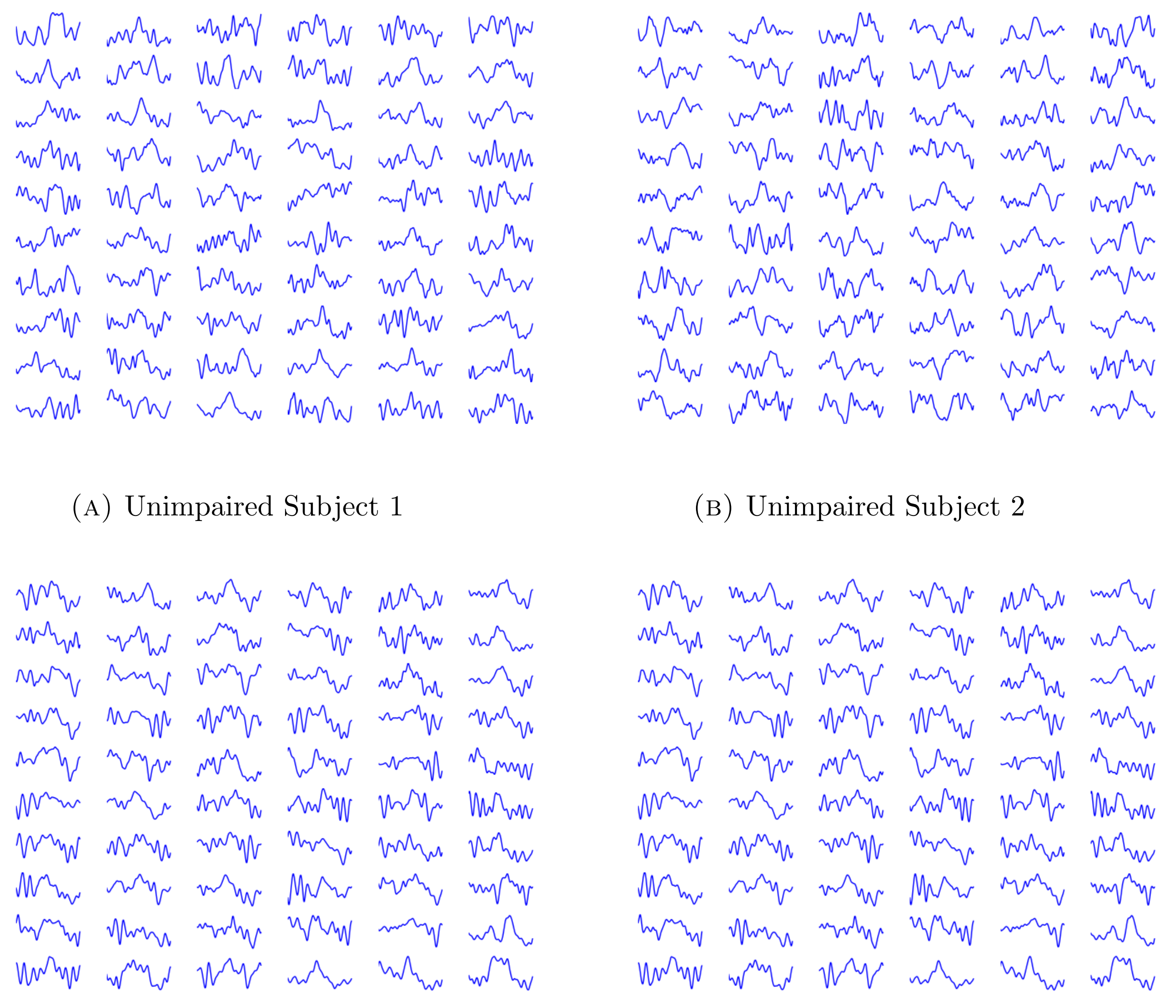
(D) Impaired Subject 2

FIGURE 4.3. Target EEG Signals after applying smoothing by Bandpass method (range between 0.5 and 30.0)

TABLE 4.1. Classification accuracies when data is Bandpass Filtered between 0.5 and 10.0. Size is number of trials per subspace

Scenario	Train Size=1	Train Size=2	Train Size=3	Train Size=4
1	**0.81**	0.78	0.77	0.78
2	**0.85**	0.78	0.74	0.73
3	0.73	**0.76**	0.72	0.72

0.5 and 10.0Hz rather than using the range between 0.5 and 30.0 Hz. Based on these results, the bandpass filter that was used on the rest of this section is between 0.5 and 30.0 Hz.



(A) Unimpaired Subject 1

(B) Unimpaired Subject 2

(C) Impaired Subject 1

(D) Impaired Subject 2

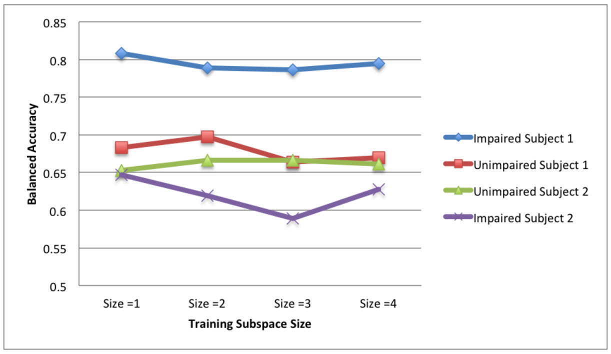
FIGURE 4.4. Target EEG Signals after applying smoothing by Bandpass method (range between 0.5 and 10.0)

TABLE 4.2. Classification accuracies when data is Bandpass Filtered Between 0.5 and 30.0. Size is number of trials per subspace

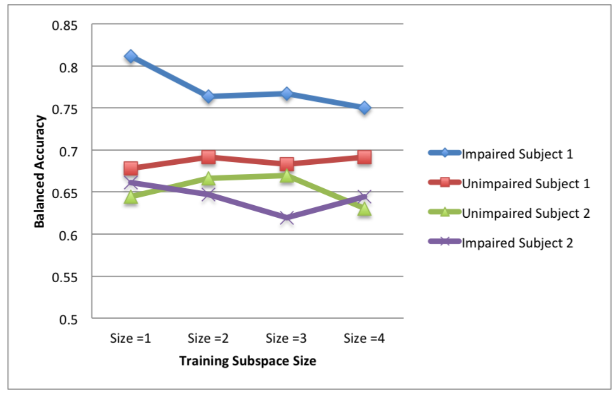
Scenario	Train Size=1	Train Size=2	Train Size=3	Train Size=4
1	**0.81**	0.76	0.77	0.75
2	**0.83**	0.79	0.78	0.72
3	0.77	**0.77**	0.74	0.76

Figure 4.5 demonstrates the classification accuracies for both preprocessing methods in the first scenario. This figure shows that smoothing by regularization tends to give better results than using the bandpass filter. In some cases, such as the case with the impaired

Subject 2, bandpass filter is better with a small difference. In Figure 4.6, the presented smoothing method tends to produce better classification accuracy with the unimpaired Subject 1 and the impaired Subject 2 in most cases, while bandpass filter tends to give better with the other two subjects. No clear trends appear in this case except with the impaired Subject 1 where the training size is one, the bandpass filter increases the classification accuracy 4% comparing with the smoothing by regularization method. In the third scenario that is presented by Figure 4.7, smoothing by regularization tends to give better results than the bandpass filter in some cases and sometimes bandpass tends to be better. As shown in Figures 4.5, 4.6, and 4.7, the classification accuracies tend to be higher with the impaired Subject 1 and tend to be lower with the impaired Subject 2. For both of the unimpaired subjects, the classification accuracies are in the middle. These results were expected since the P300 is well presented with the impaired Subject 1 and not well presented in the impaired Subject 2's signals. In addition, from these figures it is shown that either with the best case (the impaired Subject 1) and the worst case (the impaired Subject 2), having one trial to create the target and nontarget training subspaces tends to give better classification accuracies than creating the training subspaces using different number of trials. On the other hand, for the middle cases such as the unimpaired Subject 1 and Subject 2, creating the training subspaces using 2 or 3 trials increases the classification accuracies. The reason for this is adding more trials in the case that P300 is well presented doesn't increase the accuracy because more noise might be added making the classification harder. Furthermore, adding more trials to the signal that did not present P300 well made the classification accuracy worse. In the other cases where the signals are in the middle, having more trials helped because the additional trials might have one of the well presented P300 signals and made the comparison better to recognize the testing trials.

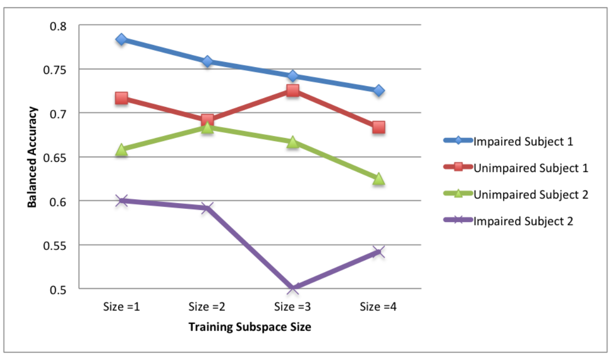


(A) Smoothing by Regularization

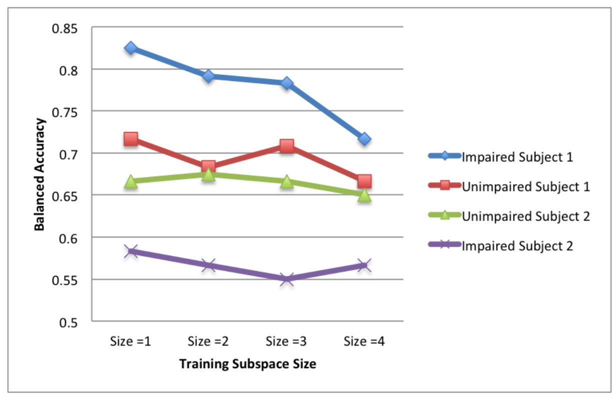


(B) Bnadpass Filter

FIGURE 4.5. Comparing the classification accuracy between the presented smoothing method and bandpass filter (Scenario 1)

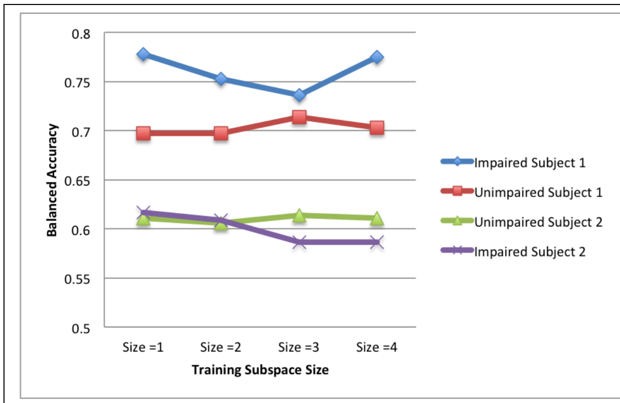


(A) Smoothing by Regularization

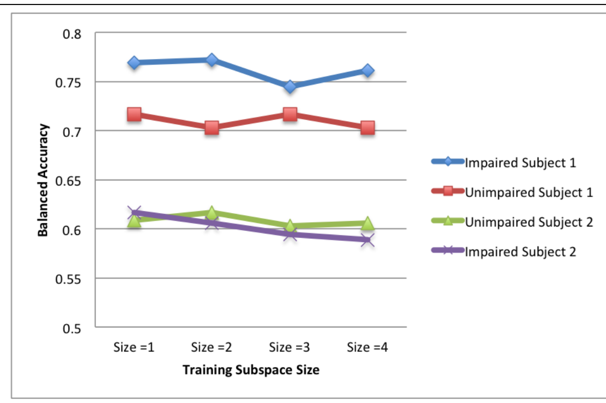


(B) Bnadpass Filter

FIGURE 4.6. Comparing the classification accuracy between the presented smoothing method and bandpass filter (Scenario 2)



(A) Smoothing by Regularization

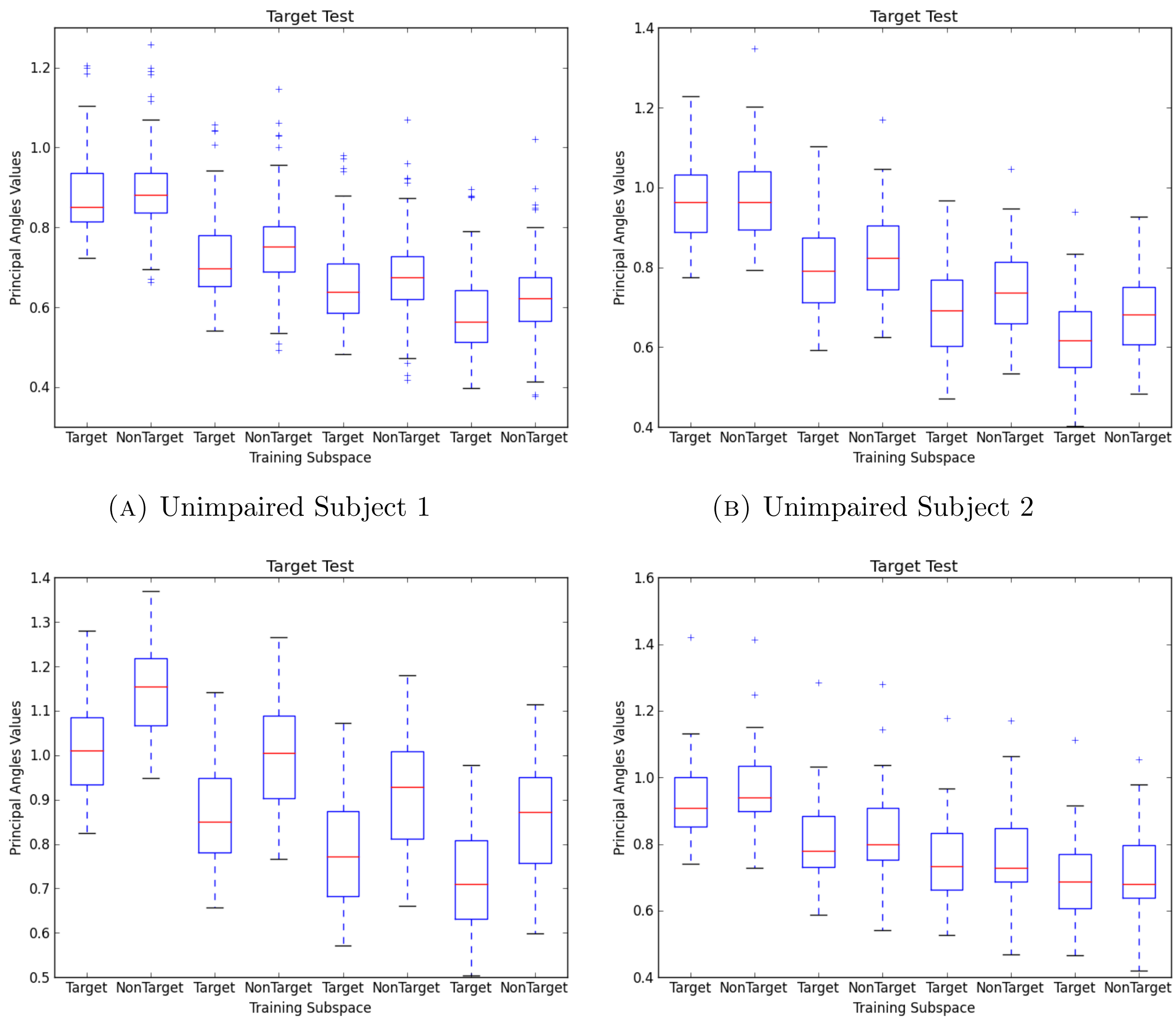


(B) Bnadpass Filter

FIGURE 4.7. Comparing the classification accuracy between the presented smoothing method and bandpass filter (Scenario 3)

Figures 4.8 - 4.19 show the values of the principal angles for both classifying the target and nontarget trials. These figures related to all of the four subjects for all of the training subspaces sizes 1, 2, 3, and 4. In all of these figures, the first two boxes are for the case where the i number of trials per subspace is 1. The number of trials is two for the third and fourth boxes. Each subspace consists of three trials with the fifth and sixth boxes, and four trials for the last two boxes. Both smoothing and bandpass filter preprocessing methods are also considered in these figures. There are several points that can be taken from these figures. First, the value of the principal angles in the case when comparing target with target is smaller than comparing target with the nontarget since in the similarity case the principal angles should be smaller to recognize the testing trials correctly. This point is the same with the nontarget testing case. In addition, adding more trials decreased the principal angle values, which means the testing and training subspaces become more similar, but not necessarily correct. As shown with the figures that are related to the impaired Subject 1, the overlap between angle ranges is not that much especially with the target testing case, which means there is a good differentiation to classify the testing trials correctly and having the highest classification accuracy. On the other hand, there is a clear overlap between angle ranges in the case with the impaired Subject 2, which makes the classification process harder. For the other two subjects the overlap is not that much compared to the impaired Subject 2. As shown in Figures 4.5, 4.6, and 4.7, Scenario 1 tends to give the best classification accuracy following by Scenario 2, and ending by Scenario 3.

This conclusion can be explained from Figures 4.8–4.19. For example with the impaired Subject 1, the overlap in the first scenario is not that much compared with the second and third scenarios. From all cases it is clear that the recognition of the target trials tends to be better than the nontarget trials since the overlap between boxes is more in the nontarget case and more variations are available in these trials.



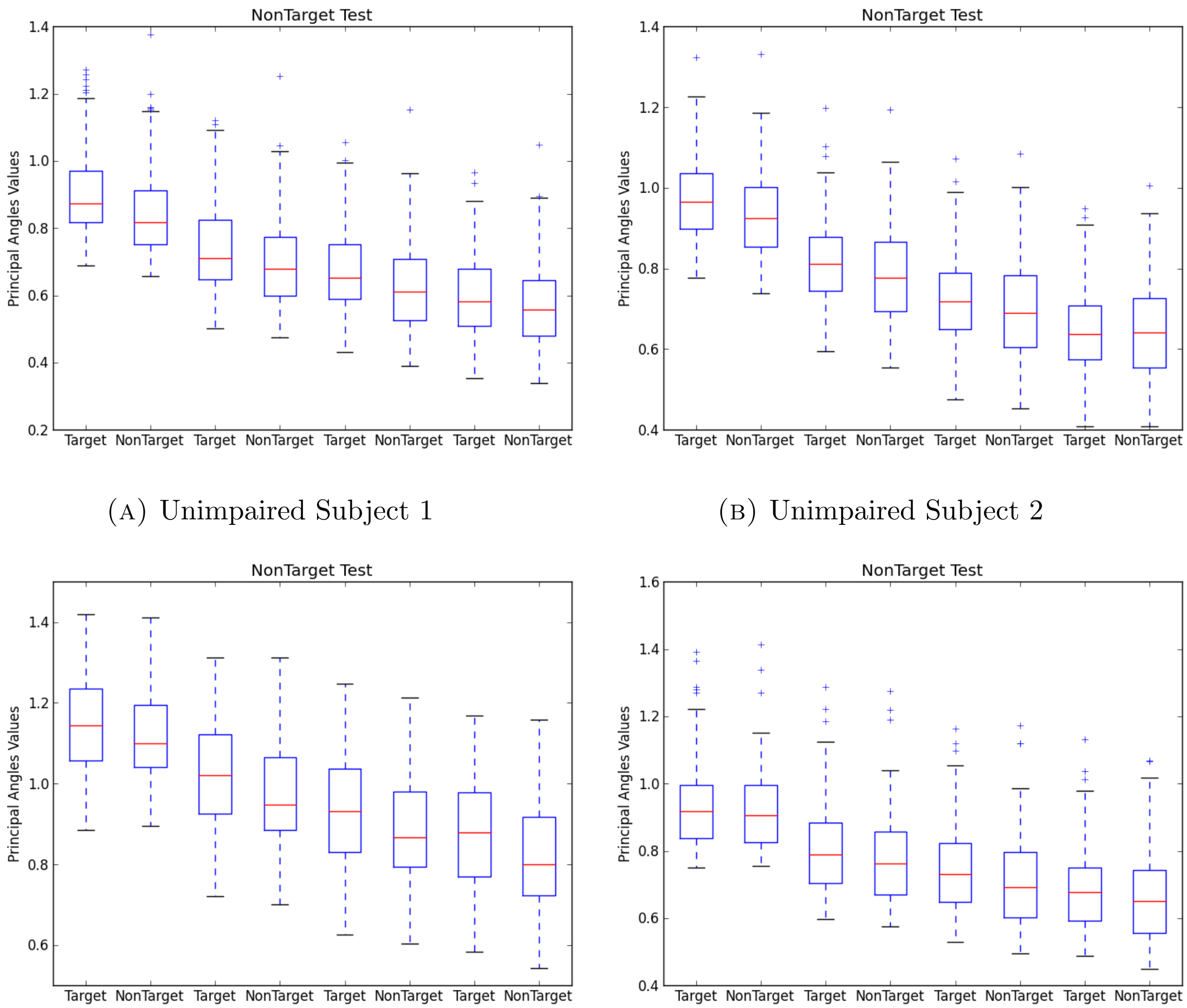
(A) Unimpaired Subject 1

(B) Unimpaired Subject 2

(C) Impaired Subject 1

(D) Impaired Subject 2

FIGURE 4.8. Principal Angle values for the first scenario for all training trial sizes



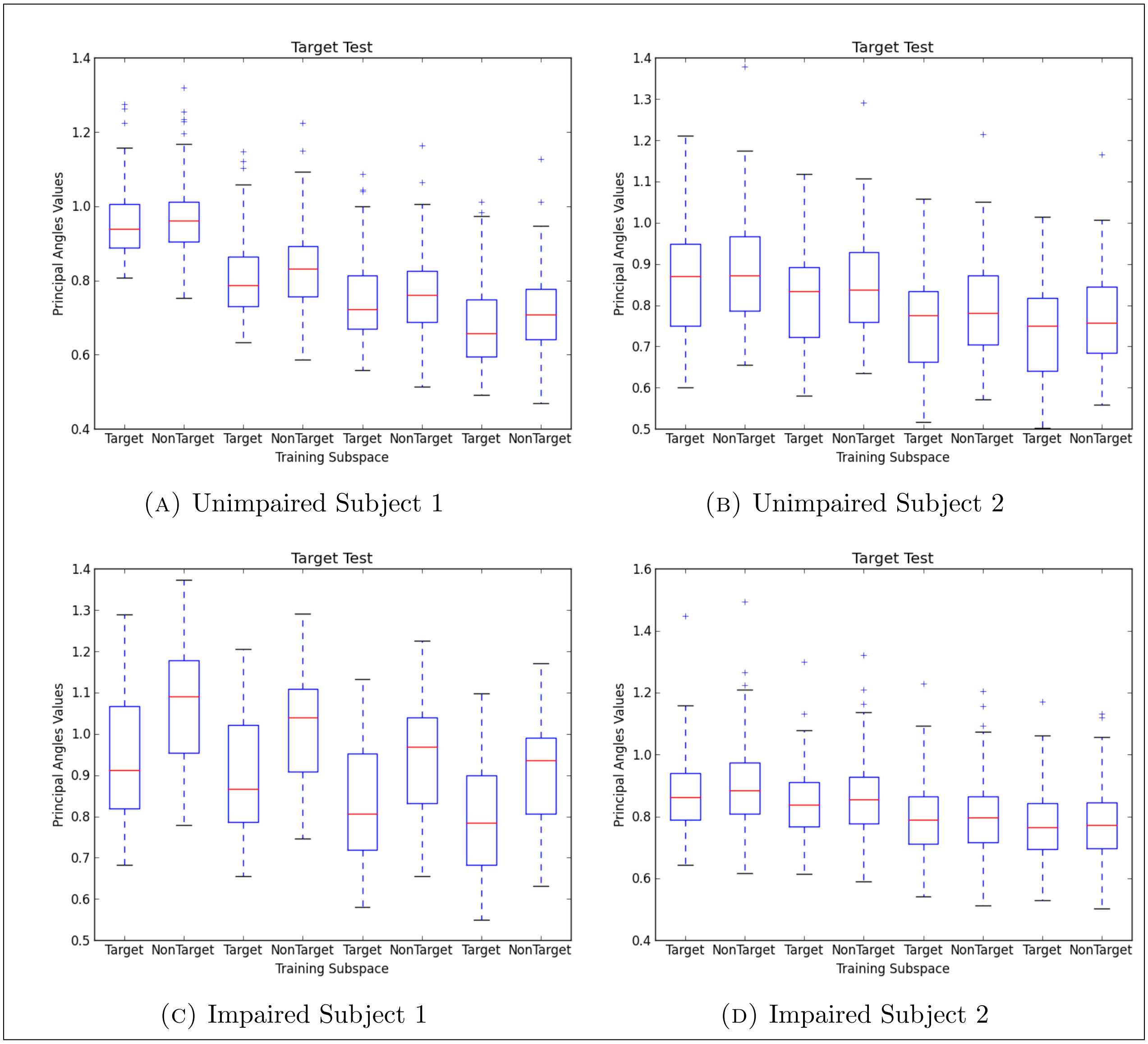
(A) Unimpaired Subject 1

(B) Unimpaired Subject 2

(C) Impaired Subject 1

(D) Impaired Subject 2

FIGURE 4.9. Principal Angle values for the first scenario for all training trial sizes



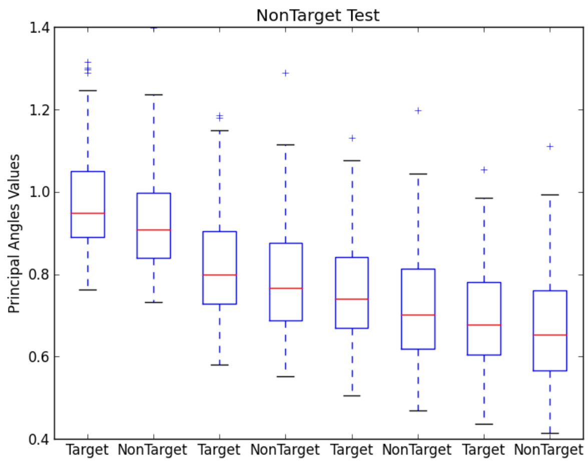
(A) Unimpaired Subject 1

(B) Unimpaired Subject 2

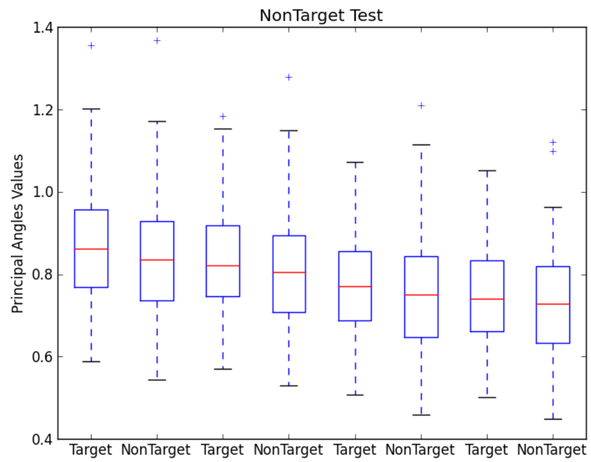
(C) Impaired Subject 1

(D) Impaired Subject 2

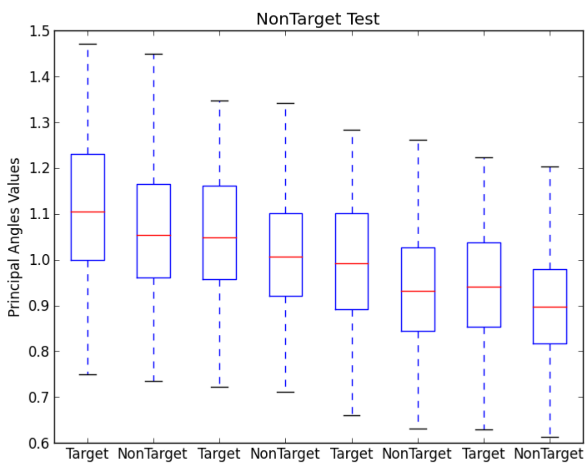
FIGURE 4.10. Principal Angle values for the first scenario for all training trial sizes using bandpass filter



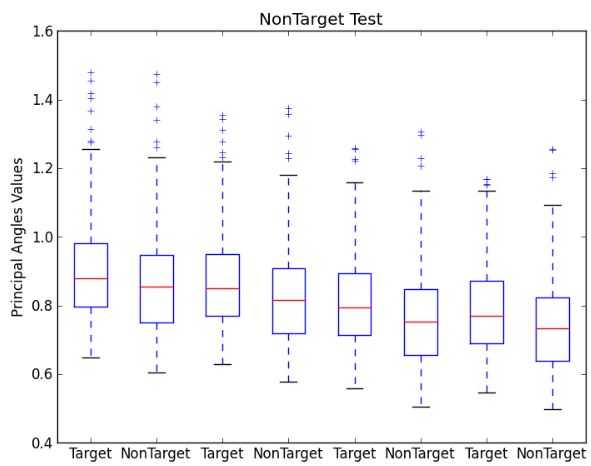
(A) Unimpaired Subject 1



(B) Unimpaired Subject 2

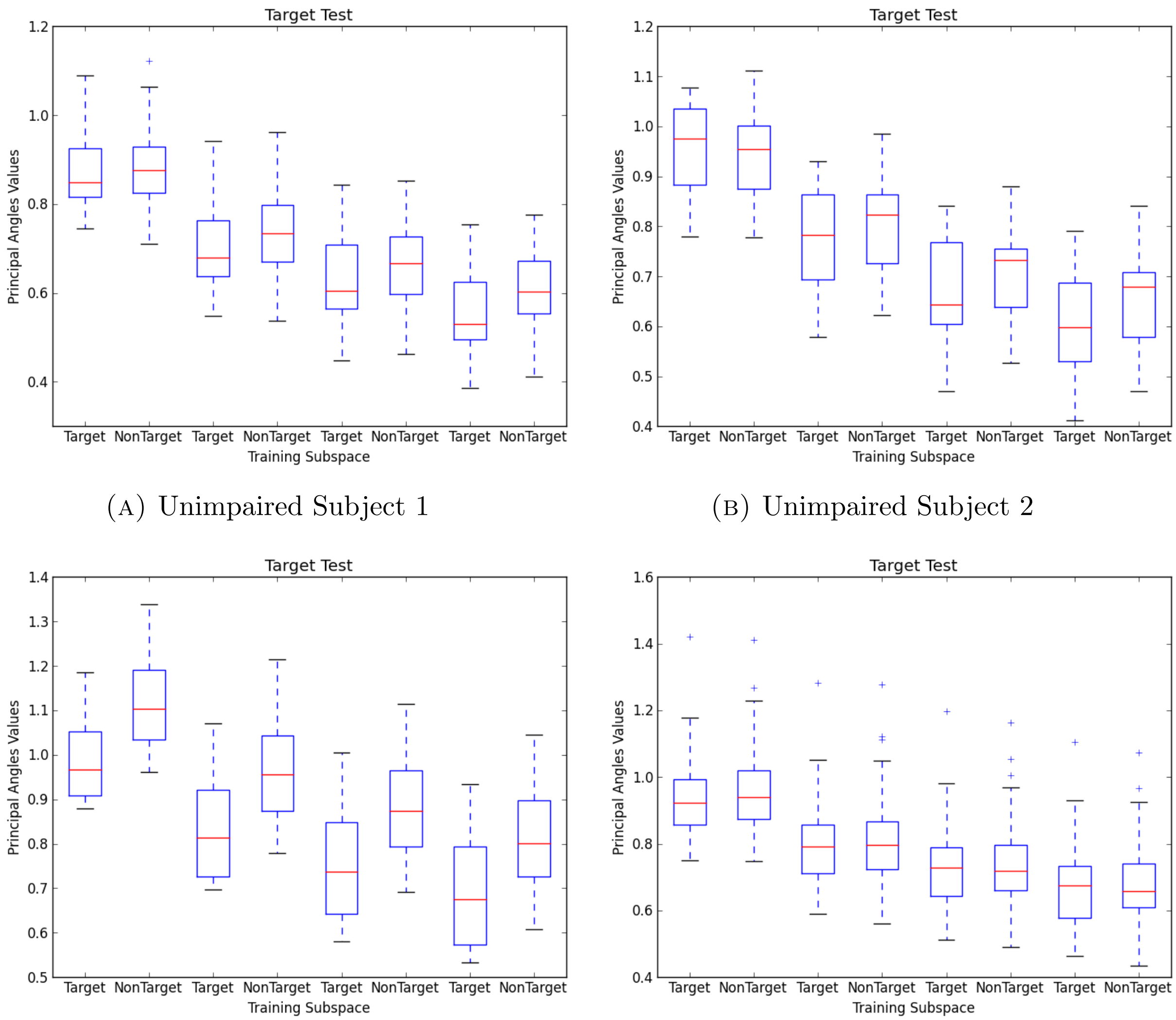


(C) Impaired Subject 1



(D) Impaired Subject 2

FIGURE 4.11. Principal Angle values for the first scenario for all training trial sizes using bandpass filter



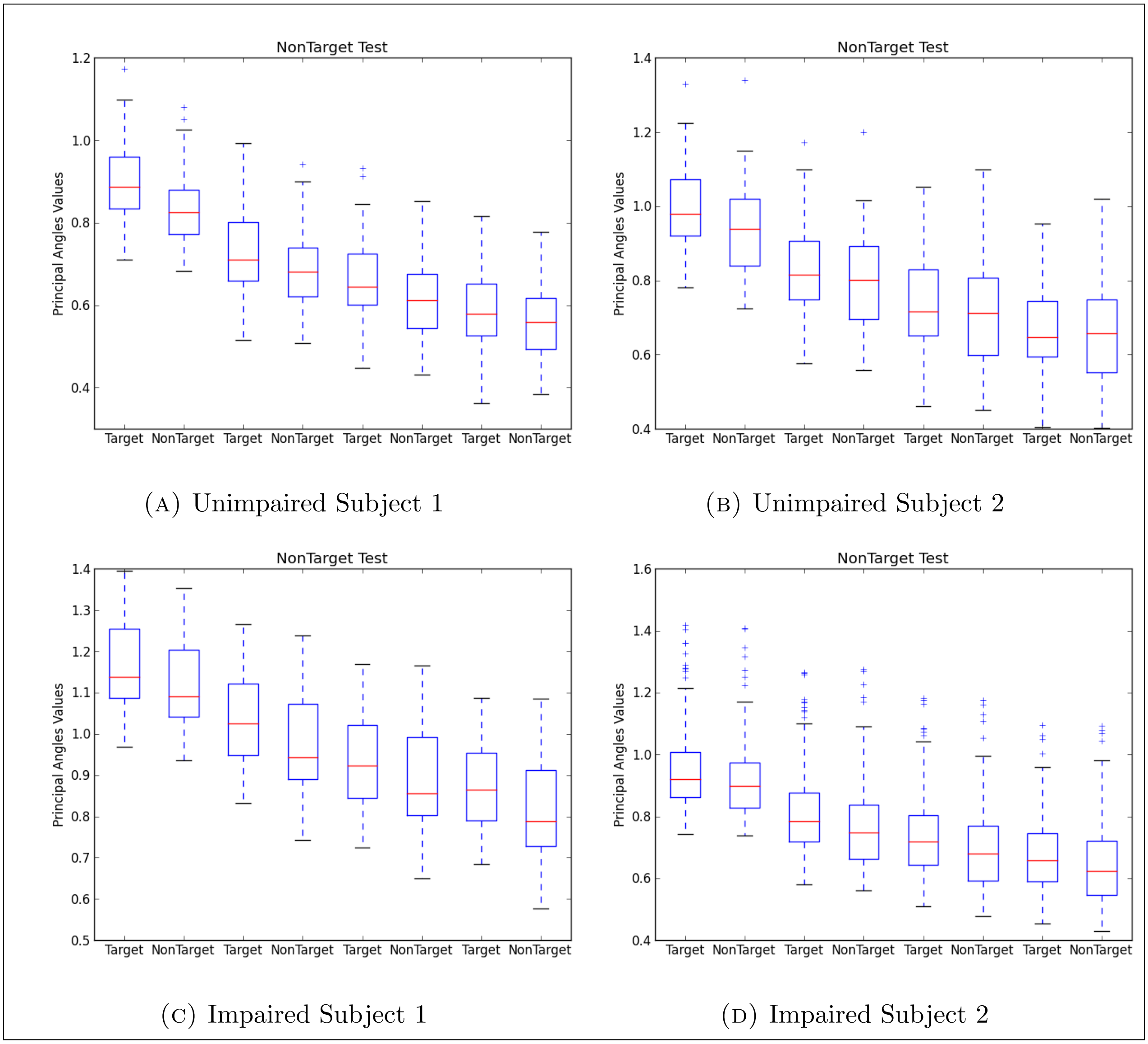
(A) Unimpaired Subject 1

(B) Unimpaired Subject 2

(C) Impaired Subject 1

(D) Impaired Subject 2

FIGURE 4.12. Principal Angle values for the second scenario for all training trial sizes



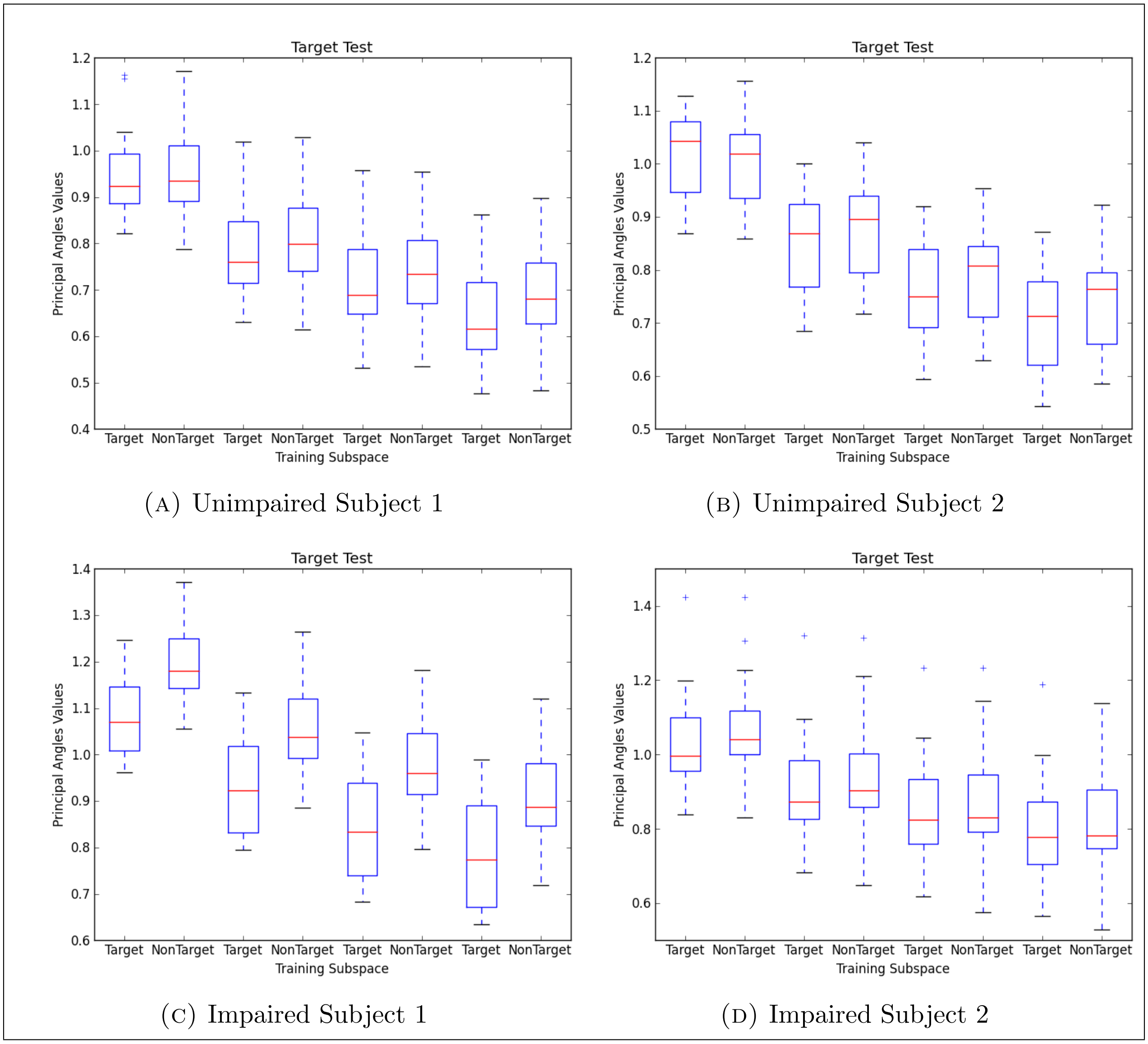
(A) Unimpaired Subject 1

(B) Unimpaired Subject 2

(C) Impaired Subject 1

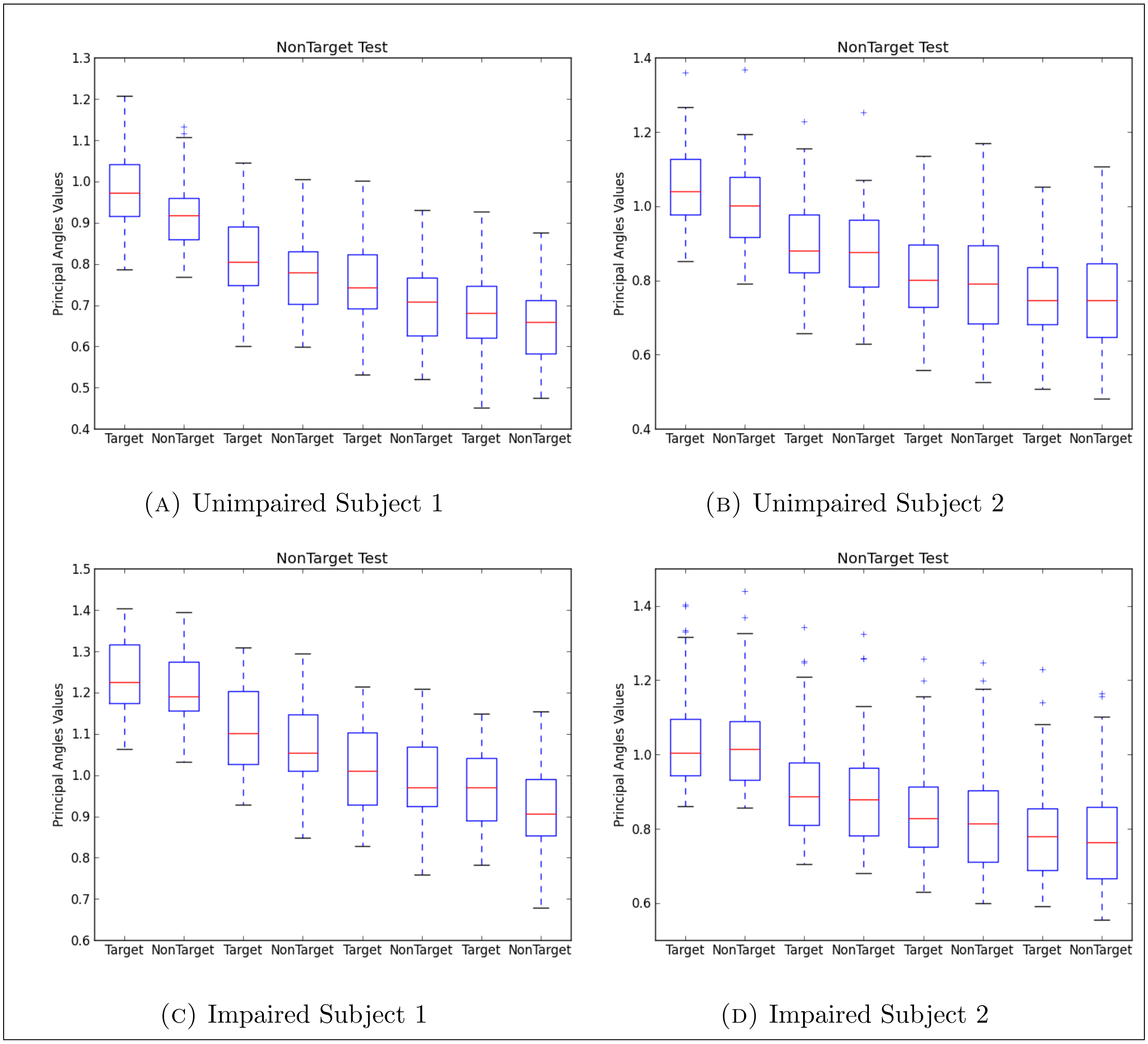
(D) Impaired Subject 2

FIGURE 4.13. Principal Angle values for the second scenario for all training trial sizes



(A) Unimpaired Subject 1
(B) Unimpaired Subject 2
(C) Impaired Subject 1
(D) Impaired Subject 2

FIGURE 4.14. Principal Angle values for the second scenario for all training trial sizes using bandpass filter



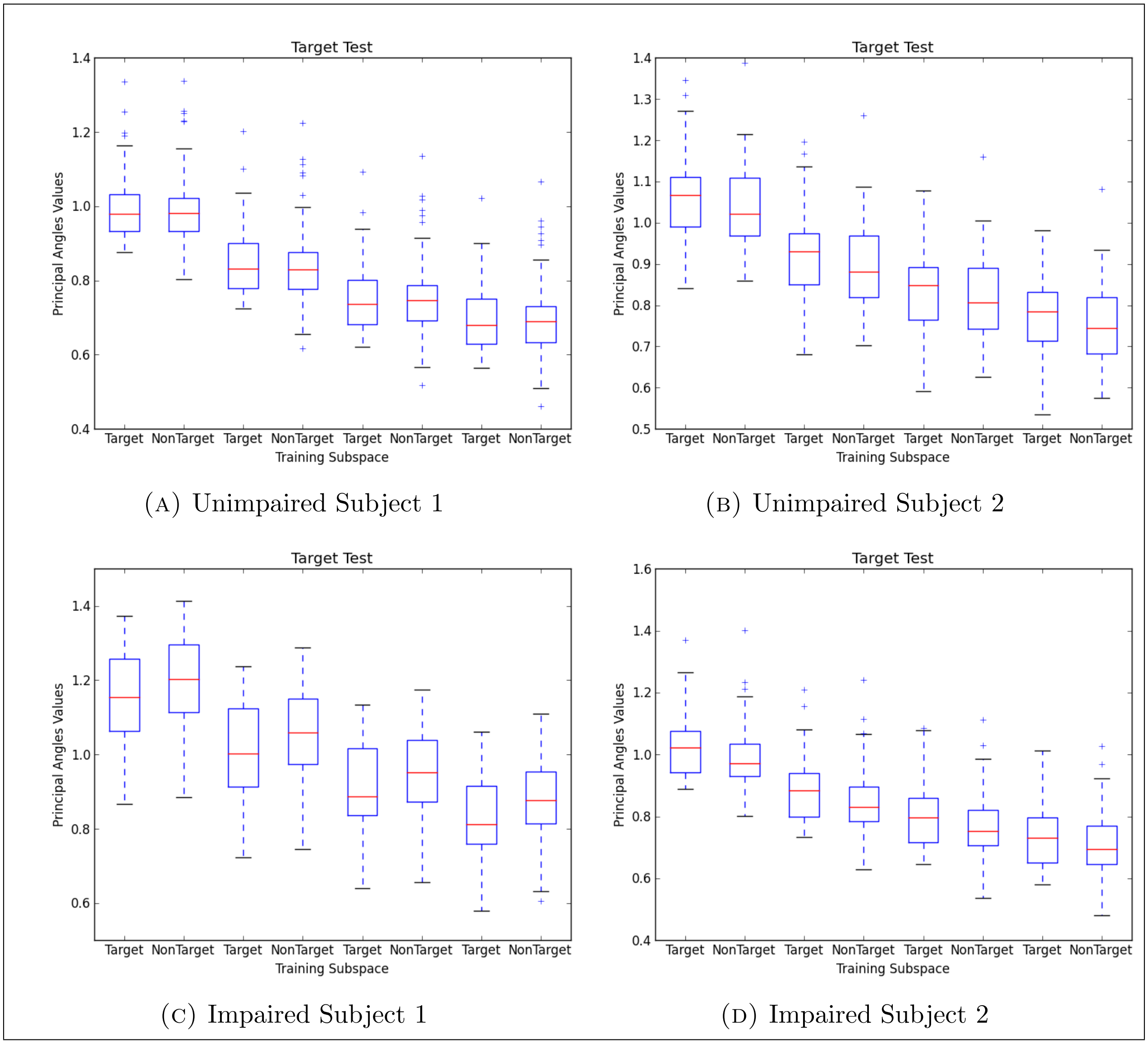
(A) Unimpaired Subject 1

(B) Unimpaired Subject 2

(C) Impaired Subject 1

(D) Impaired Subject 2

FIGURE 4.15. Principal Angle values for the second scenario for all training trial sizes using bandpass filter



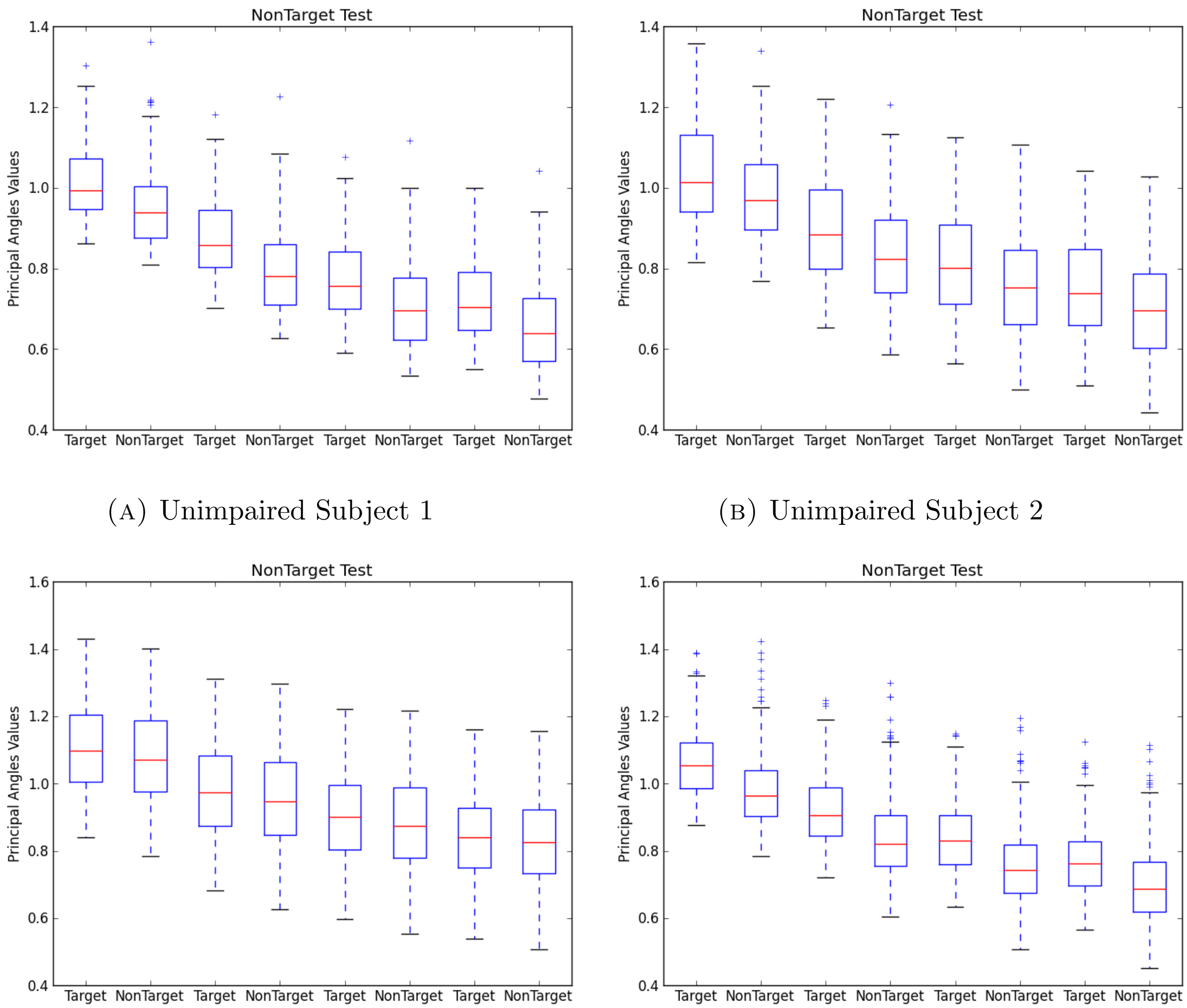
(A) Unimpaired Subject 1

(B) Unimpaired Subject 2

(C) Impaired Subject 1

(D) Impaired Subject 2

FIGURE 4.16. Principal Angle values for the third scenario for all training trial sizes



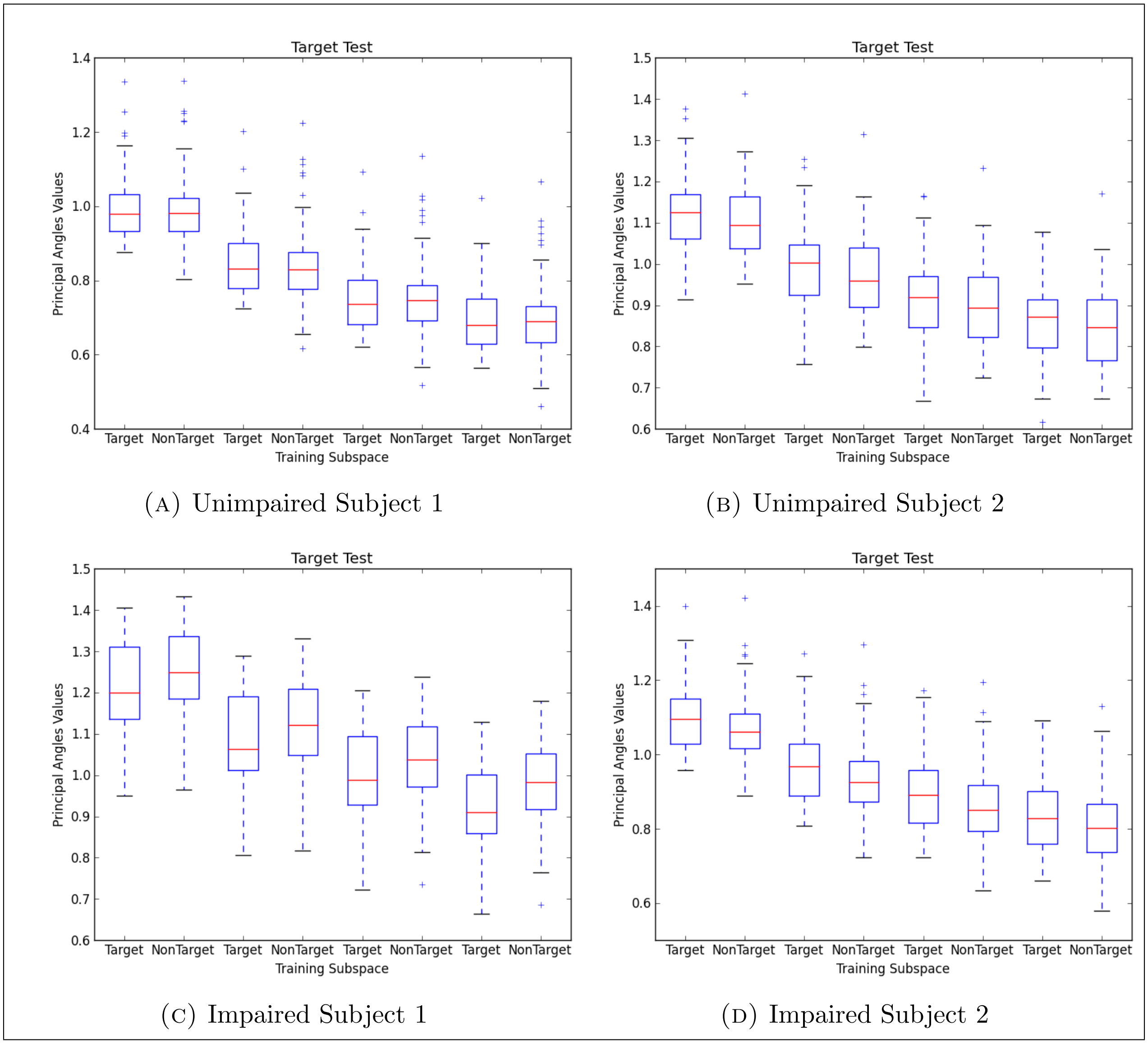
(A) Unimpaired Subject 1

(B) Unimpaired Subject 2

(C) Impaired Subject 1

(D) Impaired Subject 2

FIGURE 4.17. Principal Angle values for the third scenario for all training trial sizes



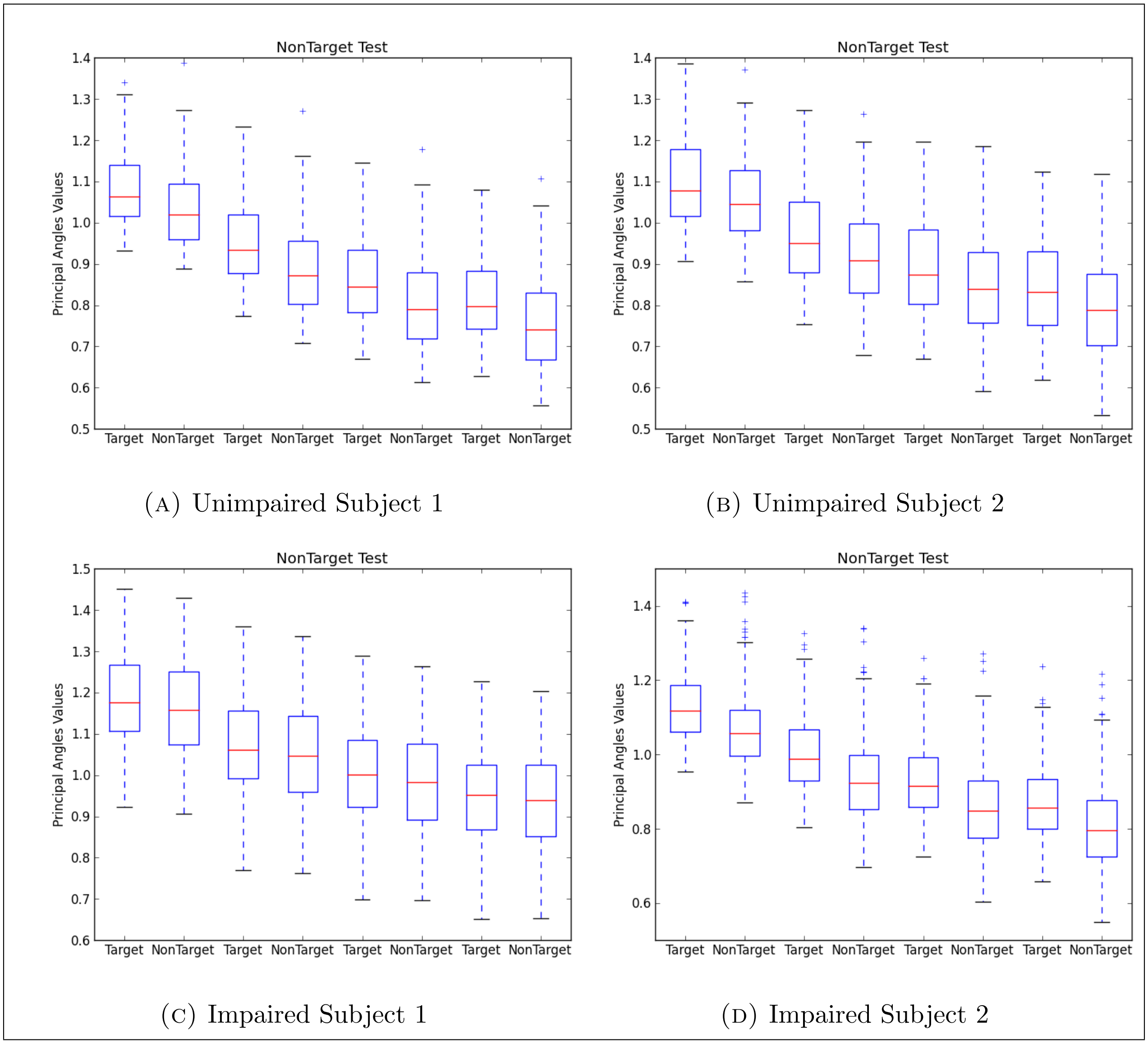
(A) Unimpaired Subject 1

(B) Unimpaired Subject 2

(C) Impaired Subject 1

(D) Impaired Subject 2

FIGURE 4.18. Principal Angle values for the third scenario for all training trial sizes using bandpass filter



(A) Unimpaired Subject 1

(B) Unimpaired Subject 2

(C) Impaired Subject 1

(D) Impaired Subject 2

FIGURE 4.19. Principal Angle values for the third scenario for all training trial sizes using bandpass filter

4.2. Outliers Removal Results

After the signals were smoothed, the classification process is implemented on both cases with and without removing the outliers. In this section the results of removing the outliers for all cases using either the minimum or the mean of the principal angles are shown. The cases that are presented in this section are when the test subspace consists of 1 trial and the training subspaces consist of 1, 2, 3, or 4 trials. All subjects were examined and the first scenario of the impaired Subject 1's outliers figures and numbers are demonstrated as an example since the best P300 presentation is shown in this subject's signals. In this scenario as mentioned before, the combination of all letter trials is 60 for the target and 180 for the nontarget. Data is divided into five different sets. Every time 48 trials are taken as target training and the remaining 12 as a test. On the other hand, 144 trials are taken as nontarget training and 36 as a testing trials. This process was implemented on all of the five disjoint sets. Tables 4.3–4.6 show the number of the target and nontarget training subspaces after removing the outliers from the training subspaces. Figures 4.20–4.25 show the training trials before and after removing the outliers based on the minimum and the mean of the principal angles. These figures illustrate the results of the first disjoint set as an example. In addition, the training sizes that are shown as example are 1 and 2. As displayed in Figure 4.20 and Figure 4.21, the number of the similarity measures is related to one principal angle only since the size of the trial is one, while with Figures 4.22–4.25 the principal angles are 2 based on the trial sizes as mentioned before. As shown in these figures, the value of the principal angles using the minimum measurement tends to be less than the mean.

The Balanced accuracy is more stable and reliable compared with the accuracy that was mentioned in the previous chapter. Figure 4.26 shows an example of the presented experiments with 2 training trials and 1 testing trial. As mentioned before, the measure

Table 4.3. Number of Training Subspaces After Outlier Removal in the First Scenario with training trial size is 1

Test No	Min PA1		Mean PA1	
	Target	NonTarget	Target	NonTarget
Test 1	33	95	31	94
Test 2	34	93	34	93
Test 3	34	96	31	96
Test 4	34	96	35	92
Test 5	30	97	32	95

Table 4.4. Number of Training Subspaces After Outlier Removal in the First Scenario with training trial size is 2

Test No	Min PA1		Min PA2		Mean PA1		Mean PA2	
	Target	NonTarget	Target	NonTarget	Target	NonTarget	Target	NonTarget
Test 1	32	94	34	96	24	106	34	104
Test 2	30	110	32	98	28	100	36	102
Test 3	30	104	32	102	30	102	28	100
Test 4	32	100	32	98	26	98	28	104
Test 5	34	104	30	110	26	104	30	94

Table 4.5. Number of Training Subspaces After Outlier Removal in the First Scenario with training trial size is 3. NT is the NonTarget.

Test No	Min PA1		Min PA2		Min PA3		Mean PA1		Mean PA2		Mean PA3	
	Target	NT	Target	NT	Target	NT	Target	NT	Target	NT	Target	NT
Test 1	30	90	33	81	30	111	36	102	27	102	36	96
Test 2	24	99	30	93	30	96	30	96	33	96	36	93
Test 3	33	93	24	81	24	84	27	96	27	105	36	102
Test 4	33	93	36	90	30	93	30	90	27	99	30	87
Test 5	33	102	30	96	27	99	33	90	27	111	33	99

Table 4.6. Number of Training Subspaces After Outlier Removal in the First Scenario with training trial size is 4. T is the Target and NT is the NonTarget.

Test No	Min PA1		Min PA2		Min PA3		Min PA4		Mean PA1		Mean PA2		Mean PA3		Mean PA4	
	T	NT	T	NT	T	NT	T	NT	T	NT	T	NT	T	NT	T	NT
Test 1	28	104	28	96	28	84	28	88	28	104	28	100	32	104	28	108
Test 2	28	104	28	96	28	84	28	88	28	96	32	104	28	104	32	100
Test 3	32	104	32	80	32	80	36	108	28	96	28	100	28	100	24	104
Test 4	36	76	28	84	32	84	28	112	28	96	32	88	32	88	32	92
Test 5	28	112	32	88	32	92	36	112	36	92	28	92	32	100	32	100

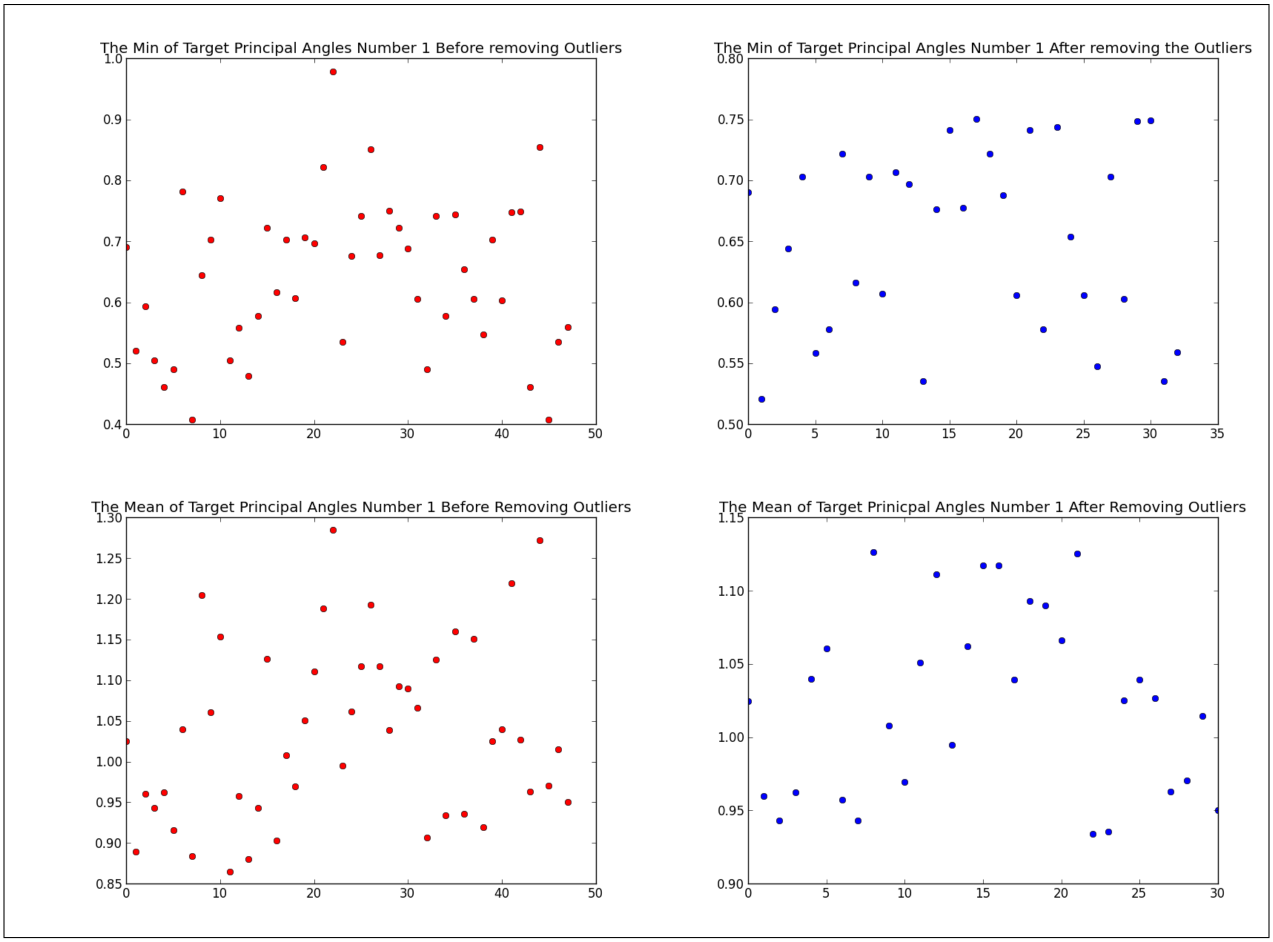


FIGURE 4.20. Removing the Target outliers. the training trials size =1

that resulted is based on the training trial sizes. All presented results in this dissertation are using the BA measurement instead of ACC. The classification accuracies are averaged over the five ways of partitioning data into training and testing sets. The accuracy is measured using BA, shown as darker gray bars, and ACC, shown as lighter gray bars, to check the sensitivity to the unequal sizes of the classes. Vertical lines on each bar show the range of minimum and maximum accuracies over the five training/testing partitions. BA gives better results than ACC for this subject in this case.

Figures 4.27, 4.28, and 4.29 show the balanced accuracy for all of the four subjects both with and without removing the outliers. In these figures, testing subspaces consist of one trial while training subspaces consist of 1, 2, 3, or 4 trials. Figure 4.27 shows the first

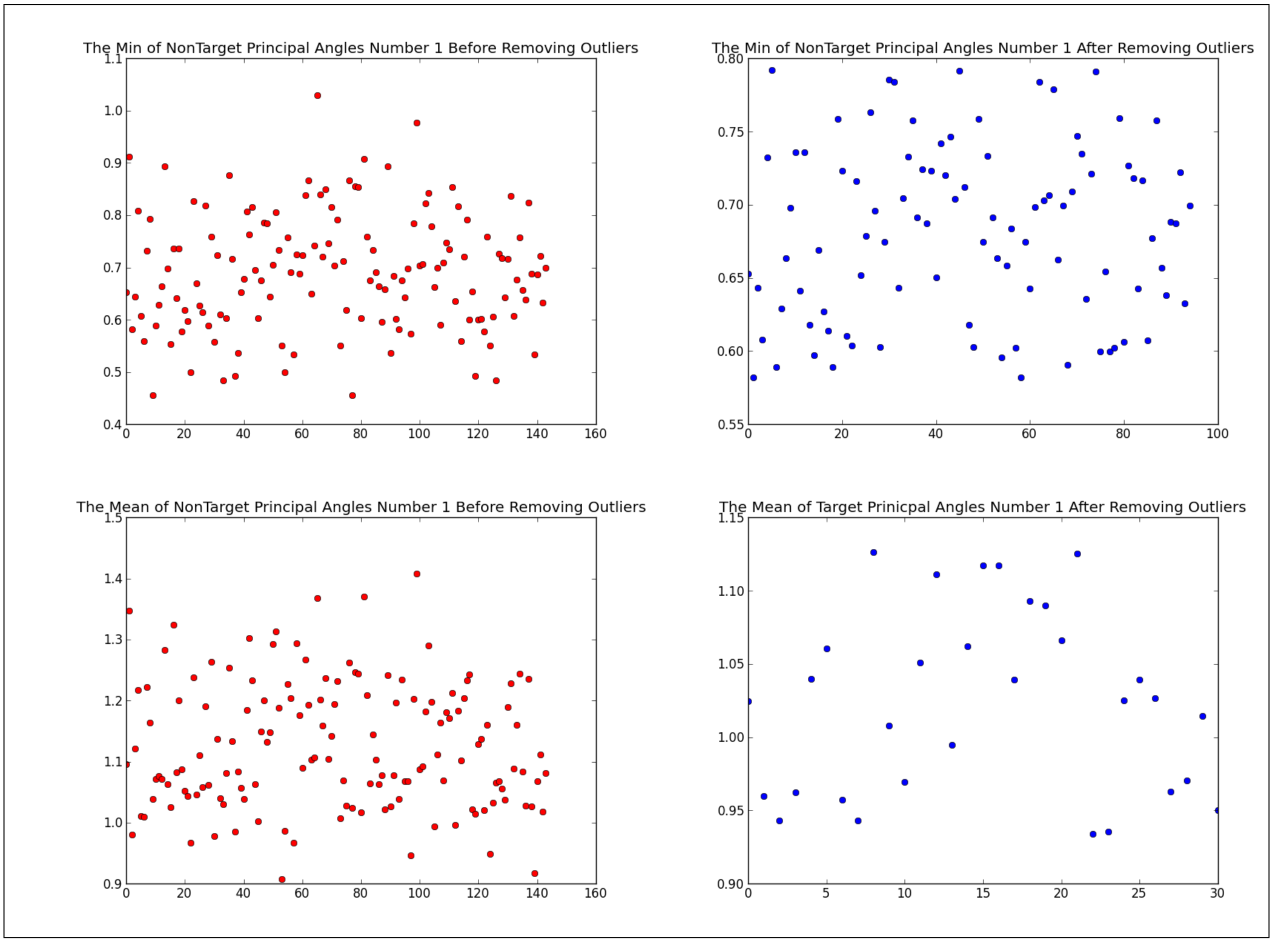


FIGURE 4.21. Removing the NonTarget outliers. the training trials size =1

scenario results, Figure 4.28 demonstrates the second scenario, and Figure 4.29 is for the third scenario. As shown in these figures, in some cases no outlier removal tends to give better results than removing the outliers. In some cases, removing the outliers increases the accuracy a small percentage. Comparing Figures 4.27 with Tables 4.3–4.6 explain the impaired Subject 1 training subspace sizes after removing the outliers in the first scenario. In the case that the training size is 1, the outliers that were removed based on the minimum of the first principal angle are less than the one that was removed based on the mean. Based on this the BA tends to be higher with the minimum measurement than the mean. The same results are reached in the case that the training size is 2. The minimum of the first principal angle tends to increase the accuracy more than the mean of the first principal angle since

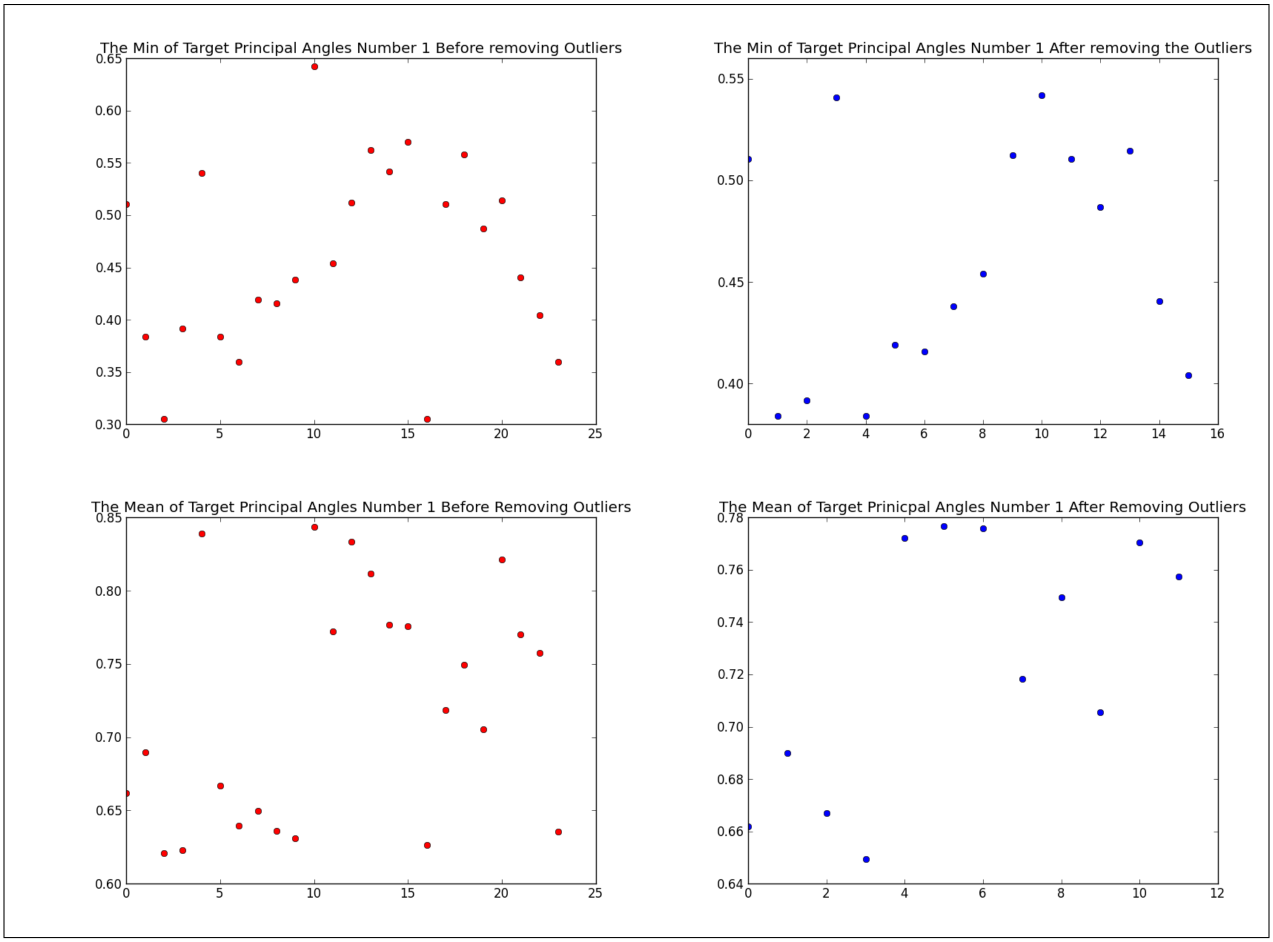


FIGURE 4.22. Removing the Target outliers based on the min and the mean of the first principal angle. the training trials size =2.

the number of the outliers that were removed is less with the minimum case. On the other hand, with the second principal angles, both methods are similar to the outliers removing number and BA results. In the case the training size is 3 and 4, the mean method tends to be better than the minimum measurement in most cases because outliers are less removed.

We speculate that an outlier can have a large effect on trial averages that is commonly used in BCI field, but an outlier as one of the trials used to define a subspace may have much less effect on the principal angles between that subspace and other subspaces.

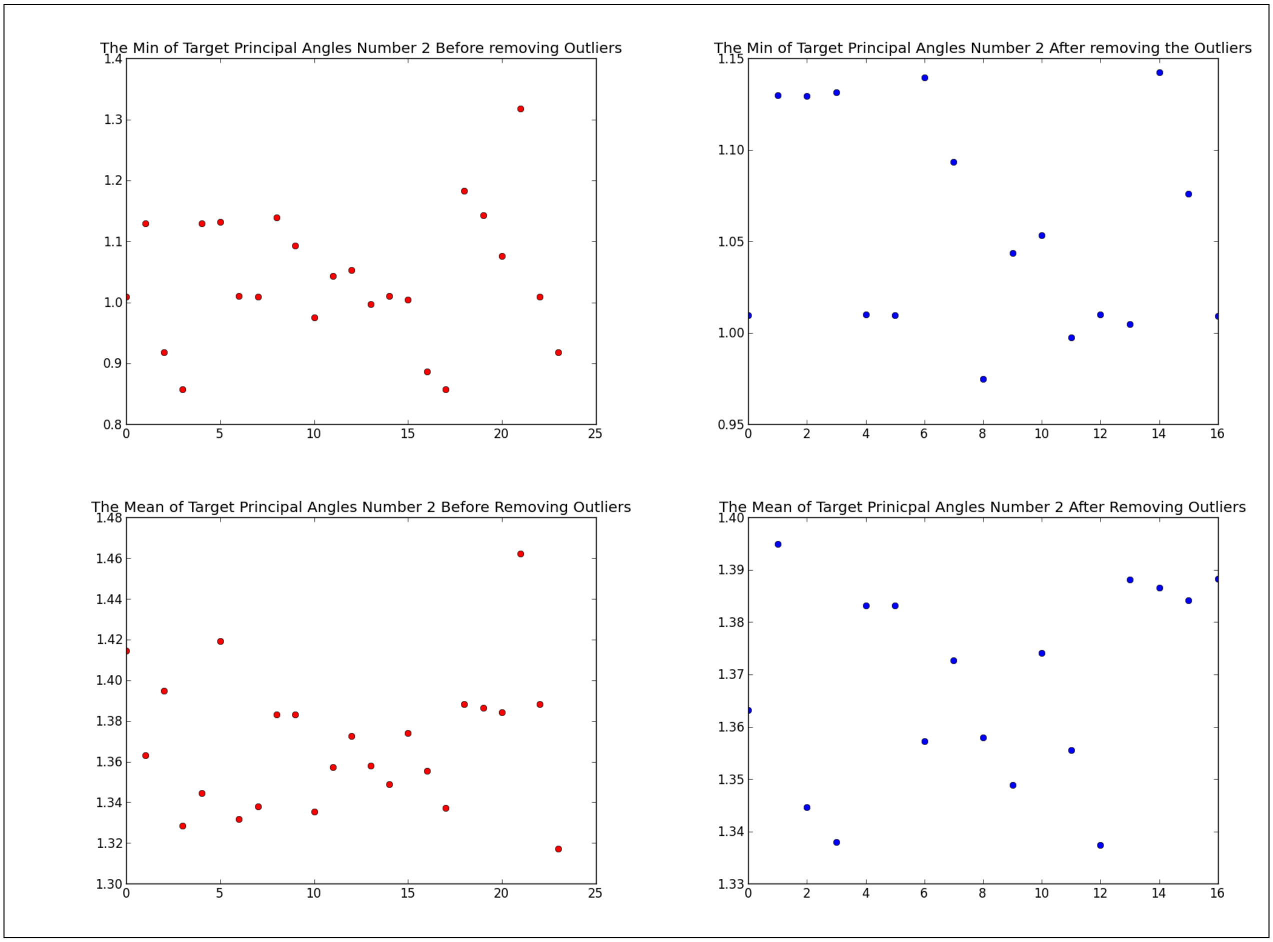


FIGURE 4.23. Removing the Target outliers based on the min and the mean of the second principal angle. the training trials size =2

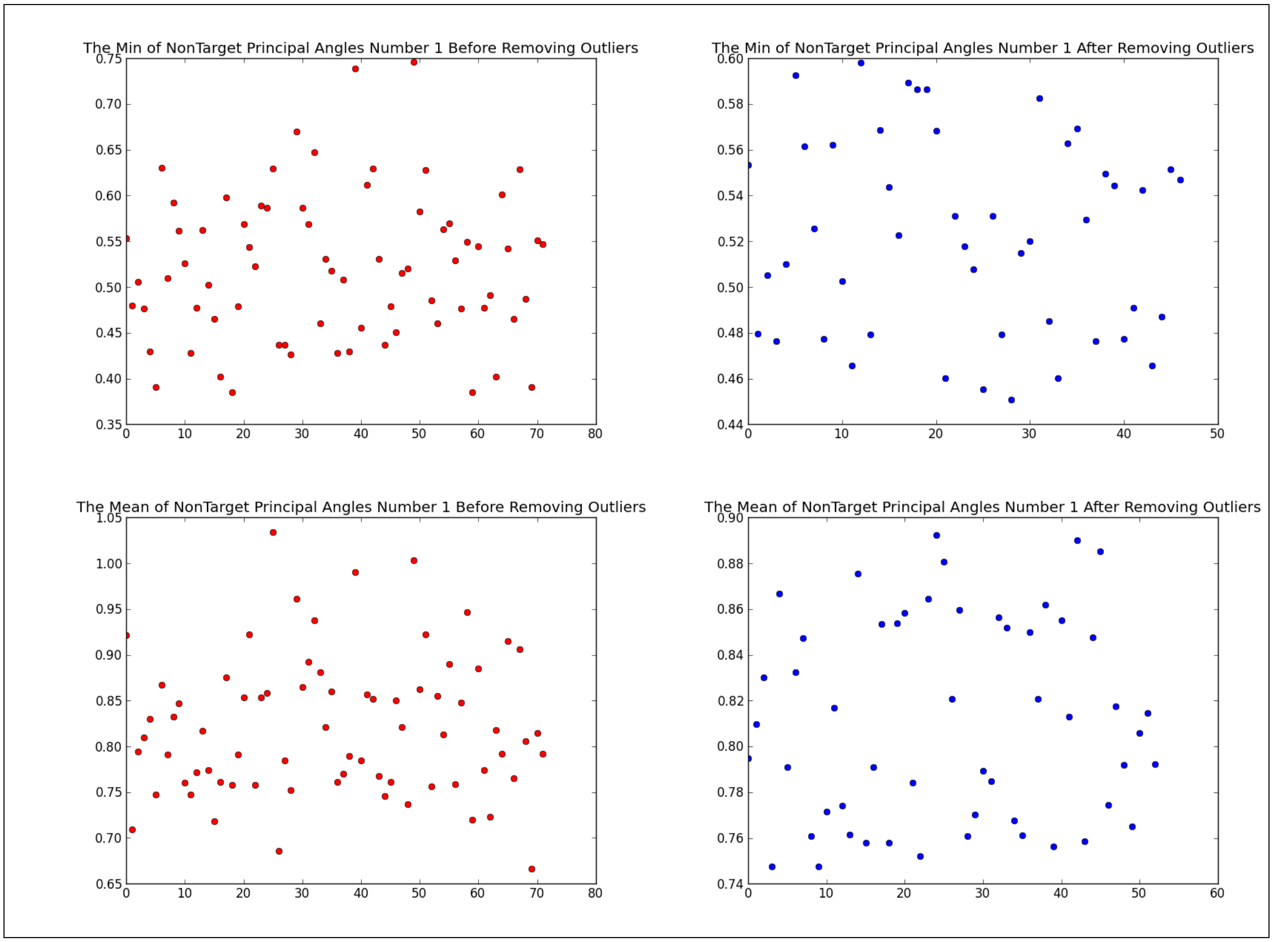


FIGURE 4.24. Removing the Non Target outliers based on the min and the mean of the first principal angle. the training trials size =2

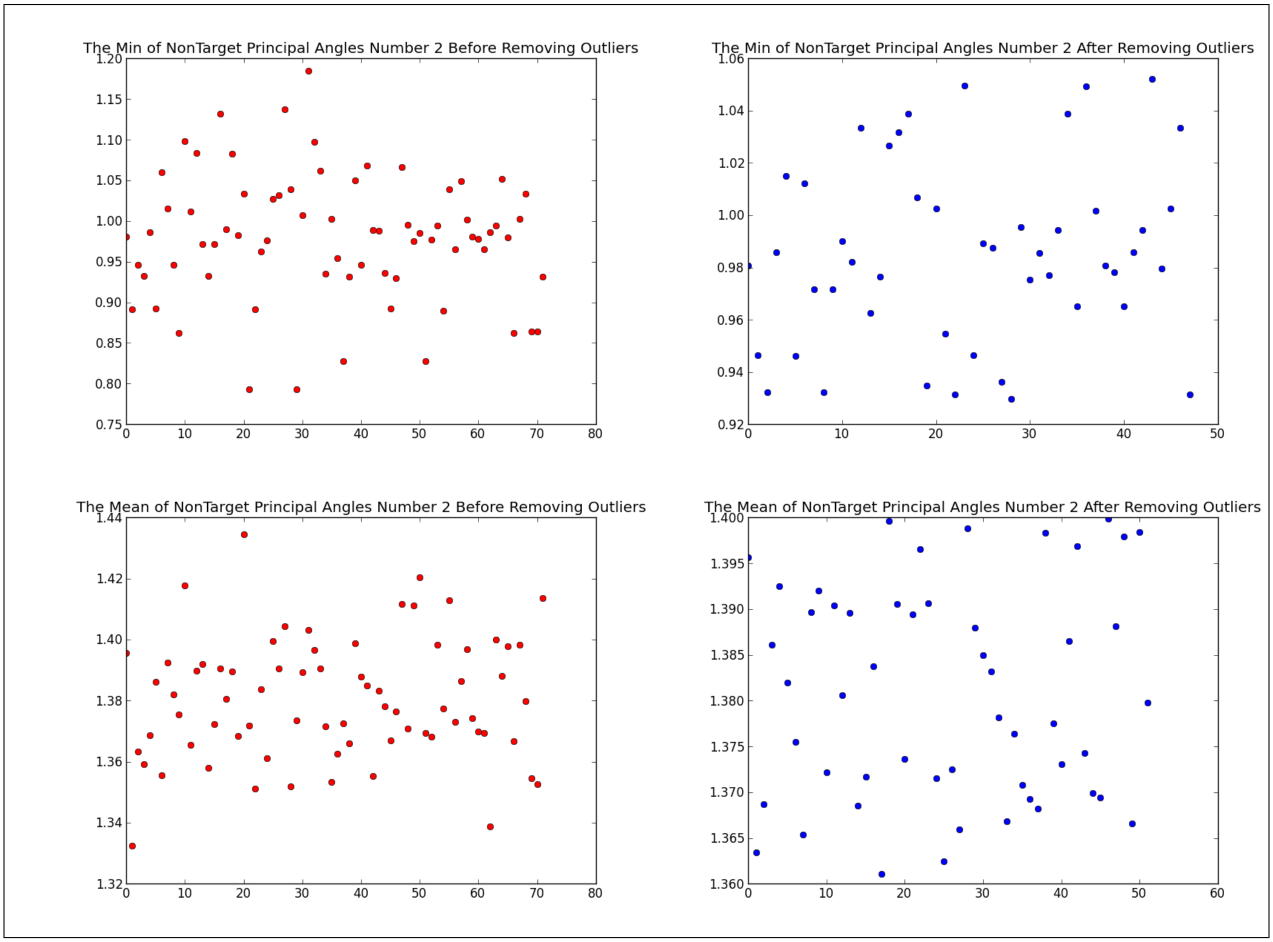


FIGURE 4.25. Removing the Non Target outliers based on the min and the mean of the second principal angle. the training trials size =2

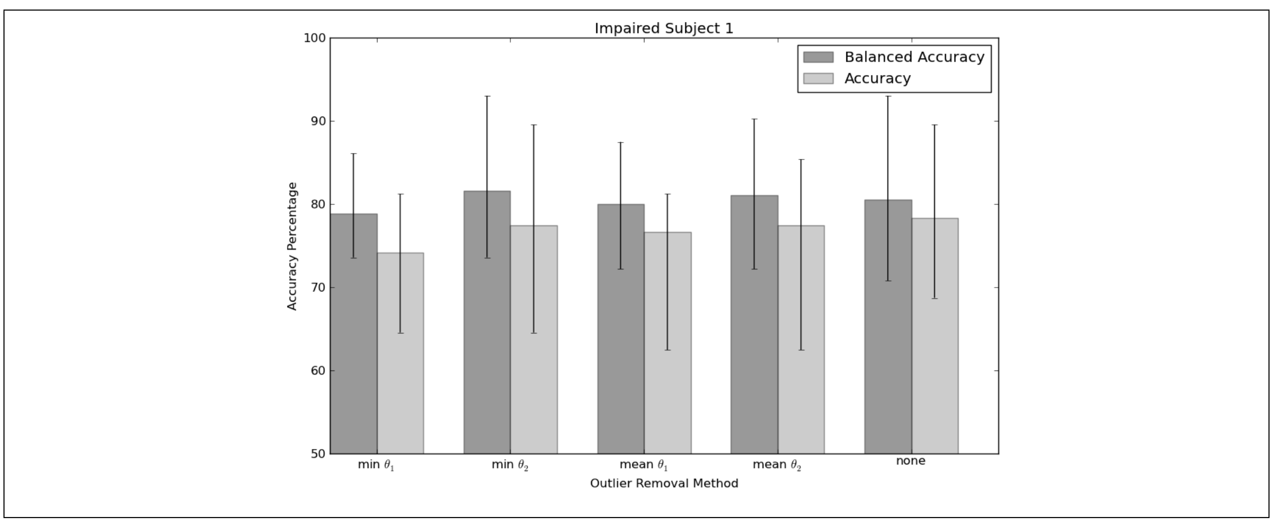
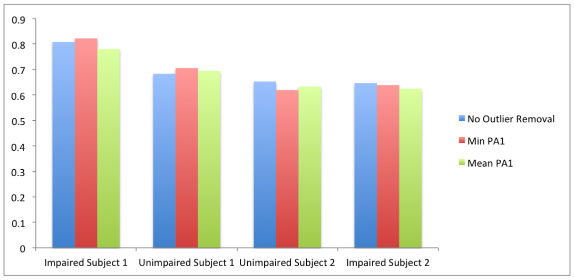
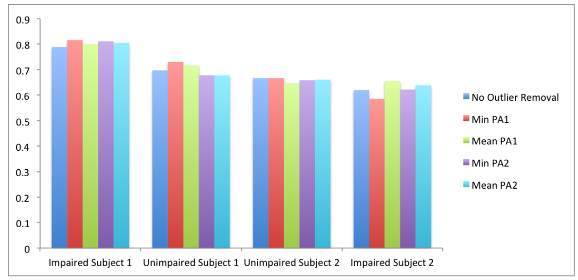


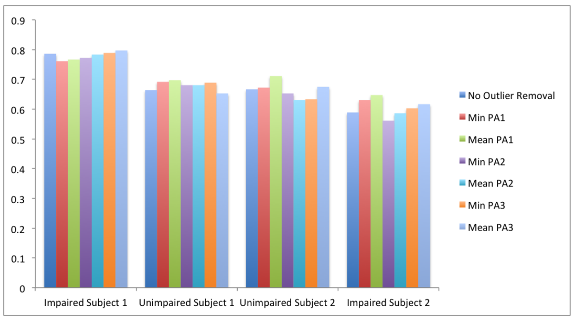
FIGURE 4.26. Classification accuracies as percentatge of test samples classified correctly.



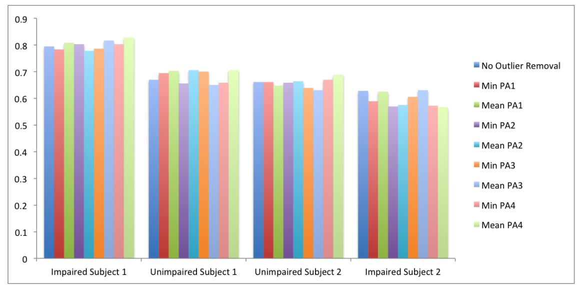
(A) Train size=1



(B) Train size=2

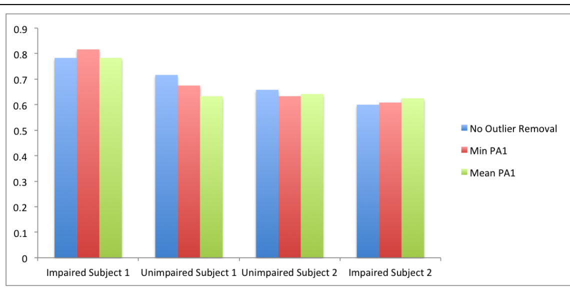


(C) Train size=3

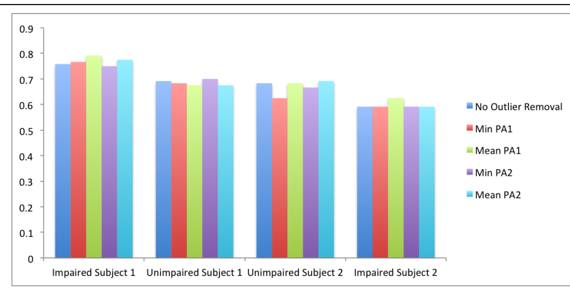


(D) Train size=4

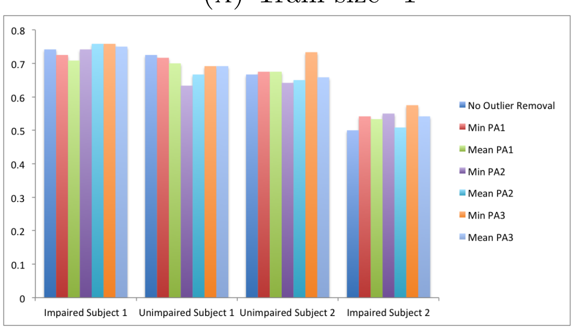
FIGURE 4.27. The Balanced classification accuracy with and without removing the outliers for the first scenario



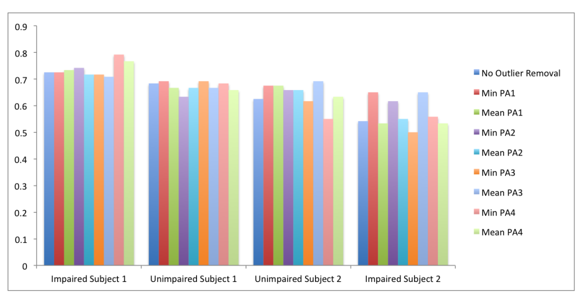
(A) Train size=1



(B) Train size=2

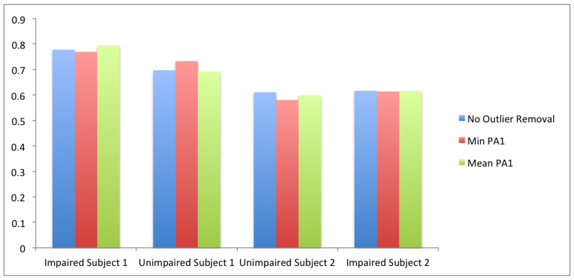


(C) Train size=3

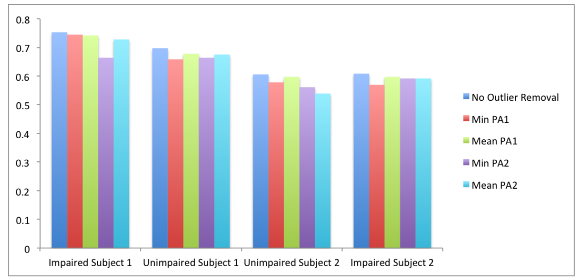


(D) Train size=4

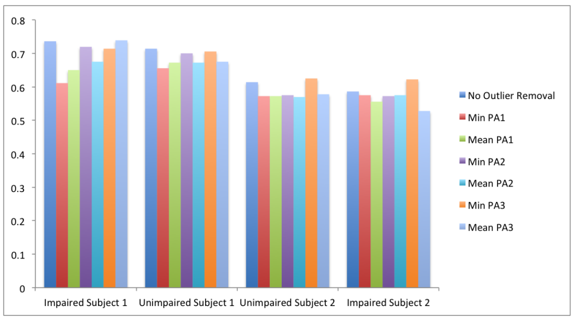
FIGURE 4.28. The Balanced classification accuracy with and without removing the outliers for the second scenario



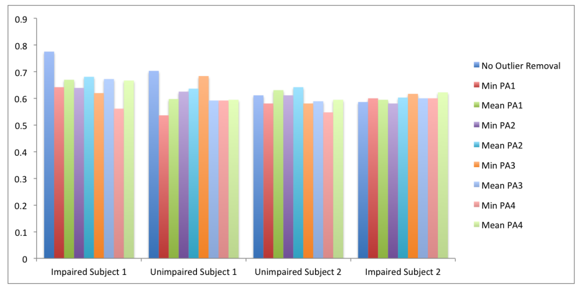
(A) Train size=1



(B) Train size=2



(C) Train size=3



(D) Train size=4

FIGURE 4.29. The Balanced classification accuracy with and without removing the outliers for the third scenario

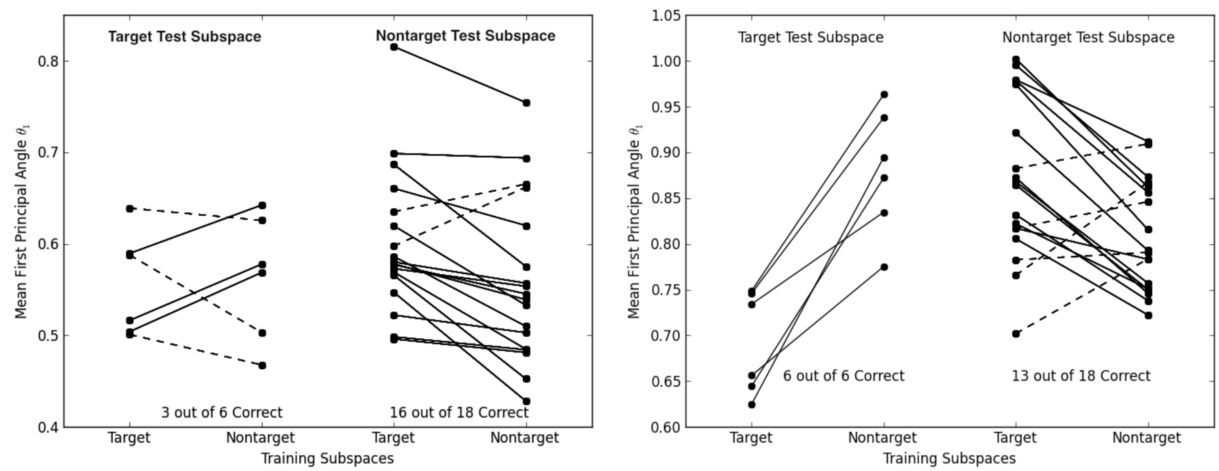
4.3. The Evaluation of the Proposed Method Using different Trial Sizes

After presenting the preprocessing methods and the outlier removal results, this section shows the results of changing not only the number of trials per training subspace, but also per testing subspace. In this section, both testing and training sizes are either 1, 2, 3, or 4 trials and the combination between these options will be presented. The results are related to the case that no outlier removal is used since, as shown in the previous section, outliers rejection did not help that much and in some cases did not help at all. All of the three scenarios are also demonstrated.

4.3.1. First Scenario.

In this scenario, 60 trials are the target and 180 trials are nontarget dividing to 48 target training trials and 12 target testing trials. On the other hand, 144 nontarget training trials and 36 as a nontarget testing trials. The values of the first principal angles between each testing subspace and training subspace are used. The mean of these angles over all target training subspaces is compared with the mean of these angles over all nontarget training subspaces. If the mean for the target subspaces is less than the mean of the nontarget subspaces, the test subspace is calculated as a target subspace, meaning one containing a P300. Otherwise, the testing subspace is classified as a nontarget.

An illustration of this comparison is shown in Figure 4.30. It plots the means of the first principal angles between each of the 24 testing subspaces (6 target and 18 nontarget) and all 96 (24 target and 72 nontarget) training subspaces in the case that training and testing subspaces consist of 2 trials as an example. Each line corresponds to these means for a single testing subspace. The endpoints of each line are the means of the first principal angles between the testing subspace and the target and nontarget training subspaces. The minimum of the two determines which class, target or nontarget, is assigned to the testing subspace.

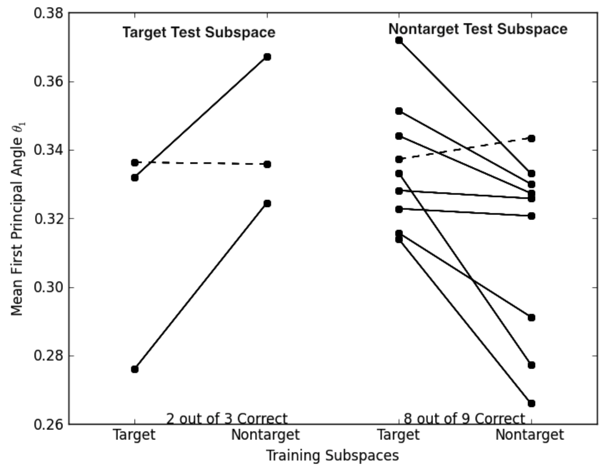


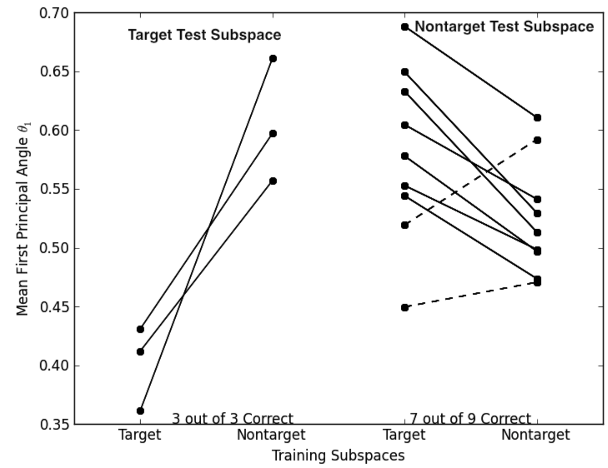
(A) Means of First Principal Angles between Test Subspaces and Training Subspaces for Unimpaired Subject 1.

(B) Means of First Principal Angles between Test Subspaces and Training Subspaces for Impaired Subject 1.

FIGURE 4.30. Classification accuracies as percentage of test samples classified correctly for the first scenario. The dashed lines show the incorrectly classified test sample

Figure 4.30a shows that 3 out of 6 with the target trials were classified correctly and 16 out of 18 with the nontarget trials .Figure 4.30b shows that all six of the target testing subspaces are classified correctly since the principal angle values are lower with the target training subspace, while 13 out of 18 nontarget testing subspaces are classified correctly. These results were reported when the method ran the first time (Test 1) and without any outliers removal. Figure 4.31 shows the results when both training and testing subspaces contain 4 trials. In this figure, only one case was misclassified for both training and nontarget cases with the unimpaired subject, while 2 cases were misclassified in nontarget subspaces with the impaired subject. One of the questions studied in this dissertation is how the number of trials for both training and testing subspaces affect the classification accuracy. Since BA is more reliable with the unequal classes size, Figures 4.32 illustrates these results



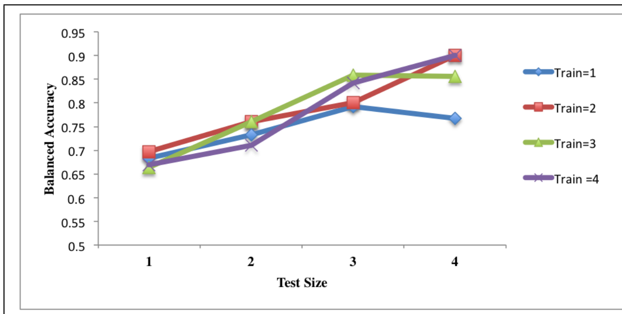


(A) Means of First Principal Angles between Test Subspaces and Training Subspaces for Unimpaired Subject 1.

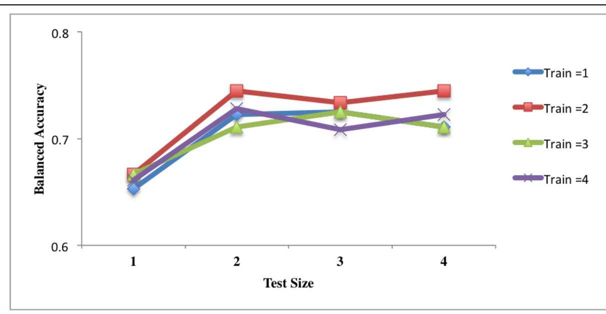
(B) Means of First Principal Angles between Test Subspaces and Training Subspaces for Impaired Subject 1.

FIGURE 4.31. Classification accuracies as percentage of test samples classified correctly using 4 trials.The dashed lines show the incorrectly classified test sample

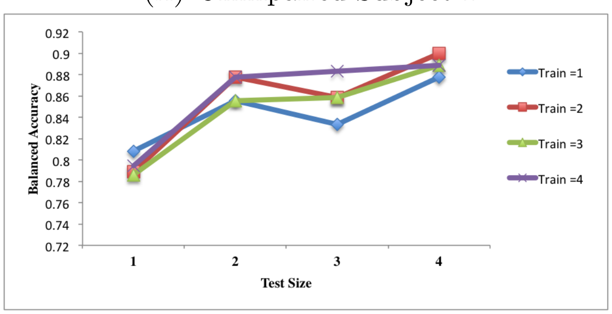
for all of the four subjects using all the different combinations of the trial sizes. As shown



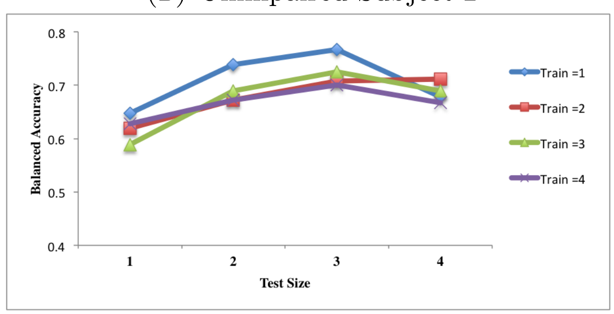
(A) Unimpaired Subject 1



(B) Unimpaired Subject 2



(C) Impaired Subject 1



(D) Impaired Subject 2

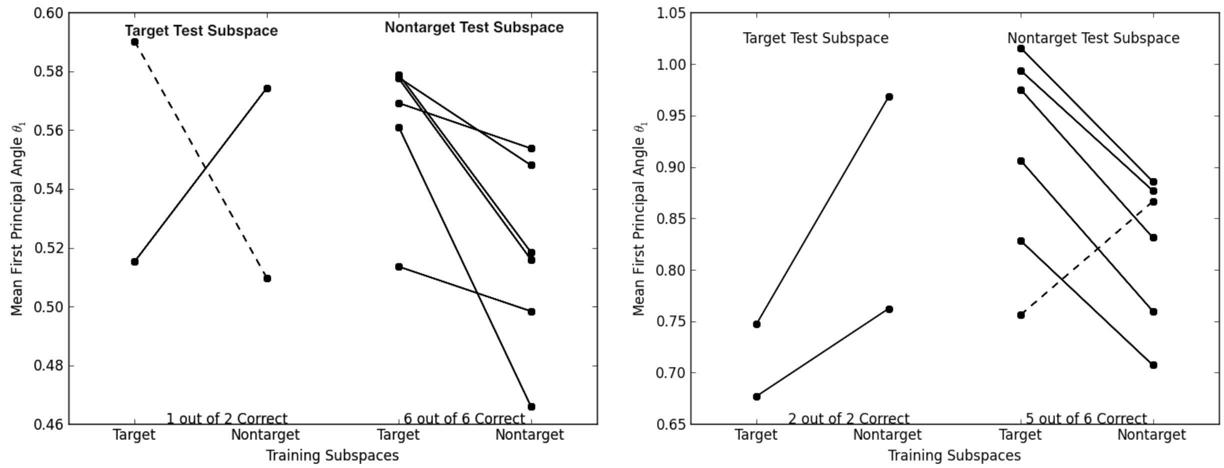
FIGURE 4.32. Classification balanced accuracies average for all trial sizes in the first scenario.

in Figures 4.32 for most cases when training subspaces created using 2 trials and testing subspaces formed using four trials, the BA values tends to be better than other cases. On the other hand for the impaired Subject 2, the best BA was reached using 1 trial only for the training subspaces and 3 trials for testing subspaces. The best accuracy was 90% for both the unimpaired Subject 1 and the impaired Subject 1, 74% for the unimpaired Subject 2, and 77% for the impaired Subject 2. Comparing between subjects, the impaired Subject 1 tends to give the best results and the impaired Subject 2 tends to give the lowest BA in most cases.

4.3.2. Second Scenario. In this scenario, the method is tested on each subject using both letters p and b as a train data and letter d as a test data. The idea from this scenario is testing the method on different data than the training dataset, which means more variations between the training and testing data since the training data subspaces do not include any of the letter-d information.

The steps in this scenario are similar to the previous one. First, the training data is smoothed. The training subspaces dimensionalities are 40 target trials and 120 nontarget, while for the testing subspaces they are 20 for the target and 60 for the nontarget. Since five testing times are applied, different testing trials are examined in each step. According to this in each testing time, 4 target and 12 nontarget disjoint trials are selected. Results demonstrate in Figures 4.33 is the caes that training and testing subspaces are created using 2 trials only.

Figure 4.33a shows that all of the nontarget trials, which are related to the unimpaired subject are classified correctly and only one target trial was misclassified. Figure 4.33b is the results that are related to the impaired subject. All target trials are identified correctly, while only one of the nontarget trials is misclassified. These results were reached by using



(A) Means of First Principal Angles between Test Subspaces and Training Subspaces for Unimpaired Subject 1.

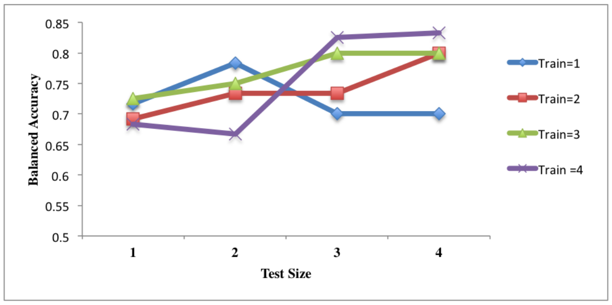
(B) Means of First Principal Angles between Test Subspaces and Training Subspaces for Impaired Subject 1.

FIGURE 4.33. Classification accuracies as percentage of test samples classified correctly for the second scenario.The dashed lines show the incorrectly classified test sample

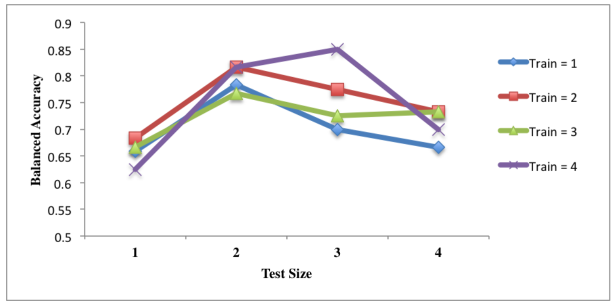
the first 4 trials from the testing target trials and first 12 trials from the testing nontarget. These results are before removing any of the principal angles outliers.

Figure 4.34 demonstrates that in most cases defining the training and testing subspaces with 4 trials tends to give better results than other cases except for the unimpaired Subject 2 gives better results when having 3 trials in the testing subspaces only and 4 trials with the training subspaces. Similar to the first scenario, the impaired Subject 1 tends to give the best results, while the impaired Subject 2 tends to be the lowest results in most cases. The highest BA for the unimpaired Subject 1 is 83%, for the unimapred Subject 2 is 85%. In addition, with impaired cases the BA for Subject 1 is 97% and it is 77% for Subject 2.

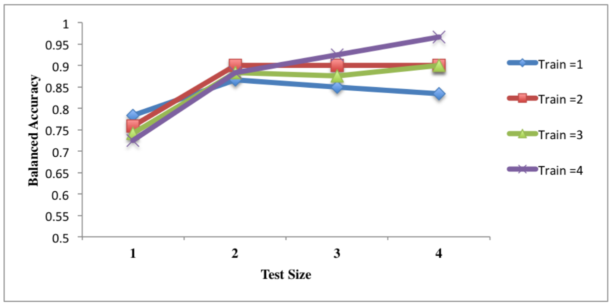
4.3.3. THIRD SCENARIO. In this section, the presented method is trained on three subjects and tested on the fourth subject. In this case, the training target trial number is 180 and 540 for the nontarget. Testing trials are 60 and 180 for target and nontarget, but because five different testing subspaces will be examined, each time 12 target trials and 36 for the



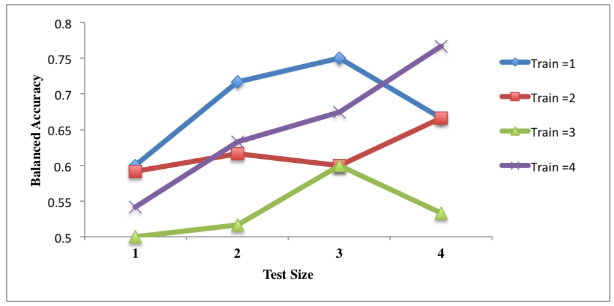
(A) Unimpaired Subject 1



(B) Unimpaired Subject 2



(C) Impaired Subject 1

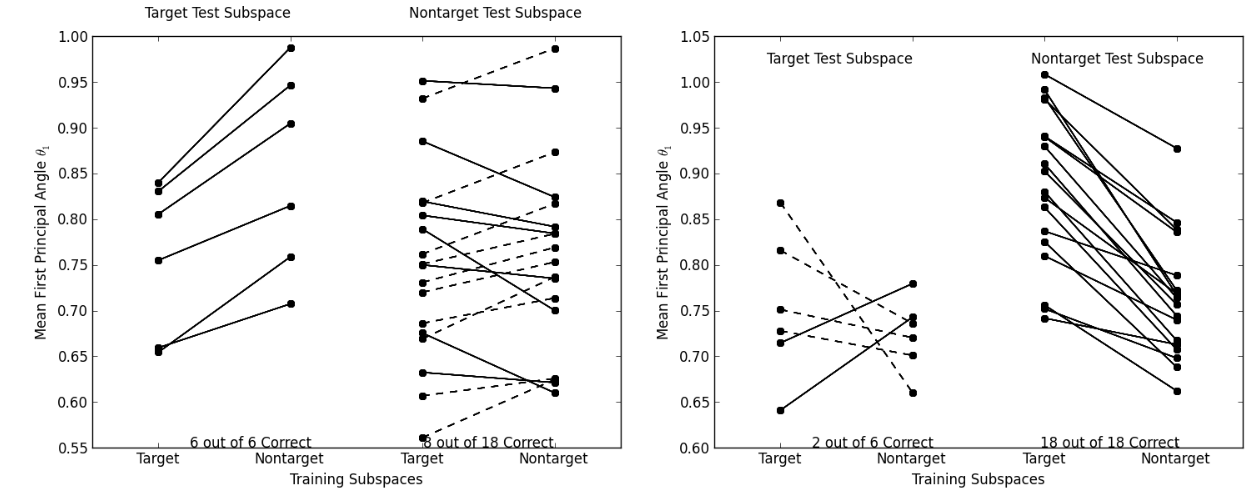


(D) Impaired Subject 2

FIGURE 4.34. Classification accuracies average for all trial sizes in the second scenario.

nontarget will be evaluated. As a start point, two subjects were tested such as unimapired Subject 1 and imapired Subject 1 with pairs trials only before evaluating the method on all subjects with all trial sizes. As shown in Figure 4.35 the method classified correctly all target trials if training was on the unimapired Subject 1, while there were four misclassification if training was on the imapired Subject 1. In addition, for nontarget trials all trials were classified correctly if the training was on the imapired Subject 1, while only 8 cases were classified correctly if the training was on the unimapired Subject 1.

Figure 4.36 demonstrates the BA for testing on all four users separately using the information from other three users. In this figure, it is shown that with unimpaired cases testing subspaces with 4 trials tends to give better results, while having 1 trial only to create the training subspaces tends to be better. Tesing on the impaired Subject 1 tends to give the highest BA and tesing on the second impaired subject tends to give the lowest BA. The best classification result when testing on unimpaired Subject 1 is 82% and 69% when testing on



(A) Means of First Principal Angles between Test Subspaces and Training Subspaces when training is on Unimpaired Subject 1.

(B) Means of First Principal Angles between Test Subspaces and Training Subspaces when training is on Impaired Subject 1.

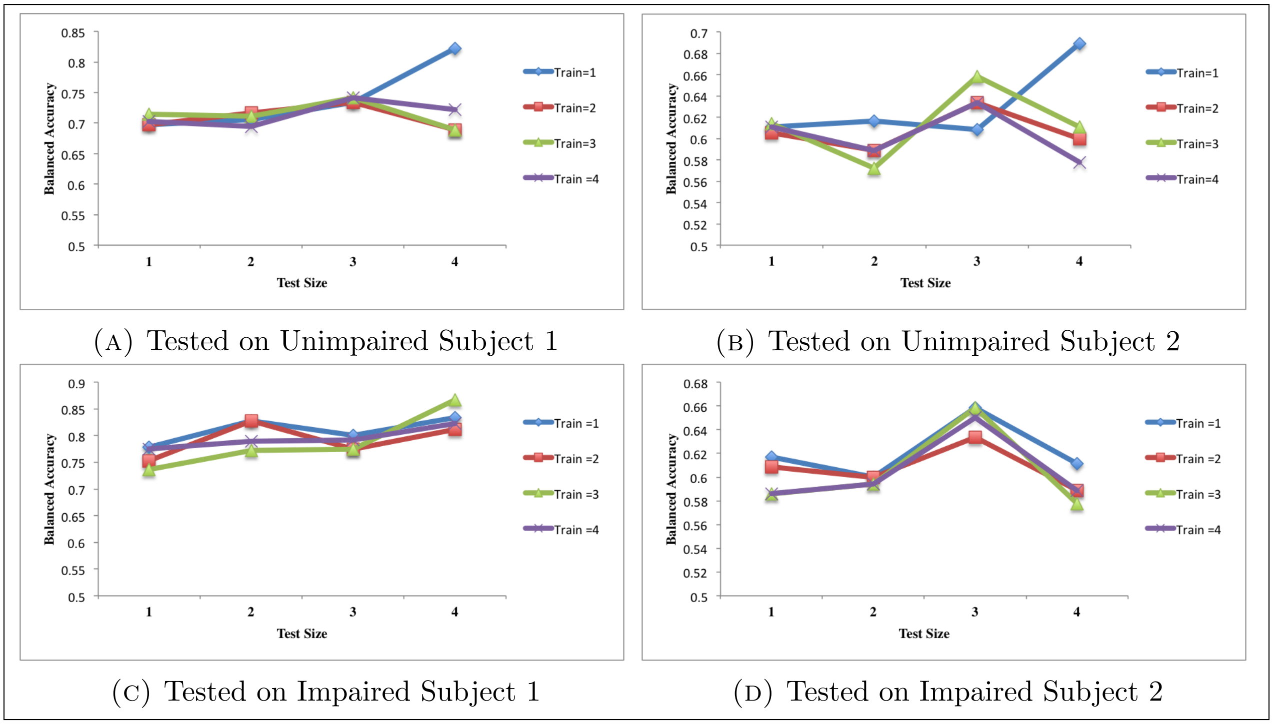
FIGURE 4.35. Classification accuracies as percentage of test samples classified correctly between subjects.The dashed lines show the incorrectly classified test sample

the second unimpaired subject. The accuracy is 87% when testing on the impaired Subject 1 if the training size is 3 instead of 1. Furthermore, testing on the impaired Subject 2 the accuracy is 66% when the testing subspaces sizes is 3 instead of 4.

As a conclusion, Figures 4.32, 4.34, and 4.36 demonstrate that using only 1 or 2 trials to create the testing subspaces tends to be more consistent than using more trials such as 3 or 4 for creating these subspaces. This point is one of the advantages that will help with online experiments in the future.

4.4. The Comparison With other Classification Method and other Recording System

Linear Logistic Regression is one of the classification methods that are used in BCI field. Tables 4.7 and 4.8 display the comparison between the presented principal angle method and this classification method. The Biosemi data is also downloaded from the website of the



(A) Tested on Unimpaired Subject 1

(B) Tested on Unimpaired Subject 2

(C) Tested on Impaired Subject 1

(D) Tested on Impaired Subject 2

FIGURE 4.36. Classification accuracies average for all trial sizes in the third scenario.

CSU BCI research group. The Biosemi system is used to record thirty two- channels EEG with a 1024 Hz sampling rate. The channel that was used in this comparison is also P4. Four more subjects were added for the G.tec recording system, two as unimpaired subjects and two as impaired. For both G.tec and Biosemi, the presented smoothing method is applied before classifying the signals using either the Linear Logistic Regression or the subspaces and the principal angles. The results are related to the experiment that is similar to the second scenario. In this case, the training was on letter-b and letter-d, while the testing was on letter-p. This experiment was chosen to match the scenario that was followed by the CSU BCI research group on their tests to reach their results. The results were obtained by creating the testing subspaces using one trial only. Figures 4.37 - 4.40 display the EEG signals that are recorded using the Biosemi system before and after smoothing preprocessing step.

Comparing between classification methods, Tables 4.7 shows that the proposed principal angle (PA) method tends to classify the testing trials that are related to the unimpaired subjects more accurately than the Linear Logistic Regression method, except for the unimpaired Subject 4. On the other hand, the Linear Logistic Regression method tends to give higher classification accuracy than the principal angle method for the impaired subjects, except with impaired Subject 3. Similar to the previous results, both classification methods give the highest BCA with the impaired Subject 1 and unimpaired Subject 2 since their EEG signal seems to display P300 representation clearer than the other two subjects. Table 4.8 shows that with the Biosemi data, the BCA results vary between the classification methods. In some cases the presented PA method tends to be better, but for other cases the Linear Logistic Regression tends to give better BCA. Dividing the data into subspaces with single or four trials only tends to give a big improvement with the impaired Subject 2 comparing with the other classification method that considered the whole training data. computation time was measured on Apple MacBook Pro with 2.7 GHz Intel Core i7.

Comparing between the two recording systems, the G.tec system needs less time to classify a testing trial to either target or nontarget than Biosemi system since the sampling rate with G.tec is less than Biosemi. For example, to classify a single trial that is related to unimpaired Subject 1, 8 seconds are needed using 2 trials for creating the training subspaces. On the other hand, 28 seconds are needed to classify a single Bioesemi trial for the same subject using also only 2 trials for developing the training subspaces.

Tables 4.7 and 4.8 demonstrate that as an average the PA method tends to give better BCA results than the Linear Logistic Regression method using either G.tec or Biosemi recording systems. Comparing between the recording systems with different sampling rates,

TABLE 4.7. Comparison between the classification accuracies g.tec g.MOBILab+

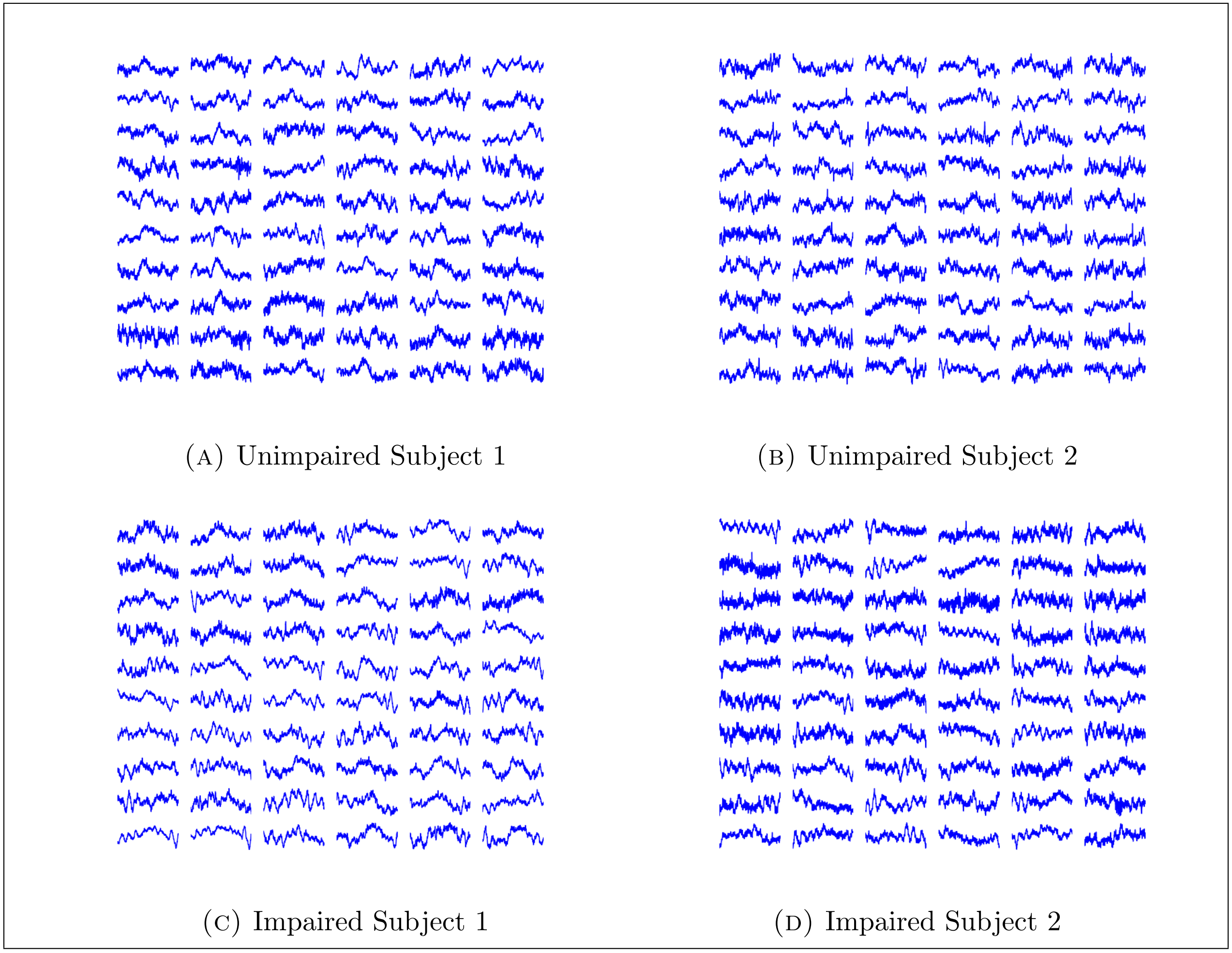
Subject No.	Linear Logistic Regression			PA		
	BCA (Training)	BCA (Testing)	λ	BCA	λ	Train Trial Size
Unimpaired Subject 1	78.8	63.3	0.0001	**71.7**	0.0001	2
Unimpaired Subject 2	78.3	51.7	0.0001	**60.8**	0.0001	1
Unimpaired Subject 3	80.0	52.5	0.0001	**60.0**	0.0001	4
Unimpaired Subject 4	73.3	**56.7**	0.0001	54.2	0.0001	3
Impaired Subject 1	94.2	**75.8**	0.0001	71.7	0.0001	3
Impaired Subject 2	76.7	**55.0**	0.0001	54.2	0.0001	4
Impaired Subject 3	80.0	55.1	0.0001	**62.5**	0.0001	2
Impaired Subject 4	72.1	**55.0**	0.0001	**51.7**	0.0001	1
Average		58.1		**60.9**		

both classification methods in most cases tend to give lower BCA using Biosemi recording system with a higher sampling rate.

This section shows that for some cases the proposed PA method tends to give better results than the Linear Logistic Regression method, but not for all cases. The following are possible explanations. First, the Linear Logistic Regression is trained on all training data, while PA method divides the training data into multiple subspaces. This concept helps to increase the BCA when the data is very noisy such as the cases of the impaired Subject 2 with G.tec system and unimpaired Subject 2 with Biosemi system as shown in both Table 4.7 and Table 4.8. The second reason is when the data is not very noisy, dividing the training data into small subspaces with training trial sizes one, three, or four helps to increase the BCA results. This thought is clearly presented with unimpaired Subject 1 and 2 using G.tec system and with unimpaired Subject 1 and impaired Subject 2 using Biosemi data. Figure 4.39 and Figure 4.40 display that dividing the training subspaces into small sets, calculating the principal angles between testing trials and these sets, and finding the matching set based on the smallest angles improves the BCA specially with the time between 500 ms and 600 ms.

TABLE 4.8. Comparison between the classification accuracies Biosemi ActiveTwo

Subject No.	Linear Logistic Regression			PA		
	BCA (Training)	BCA (Testing)	λ	BCA	λ	Train Trial Size
Unimpaired Subject 1	81.8	57.5	0.0001	**60.8**	0.0001	2
Unimpaired Subject 2	80.0	**63.3**	0.0001	60.8	0.0001	4
Impaired Subject 1	98.8	**74.2**	0.0001	63.3	0.0001	3
Impaired Subject 2	75.4	47.5	0.0001	**65.0**	0.0001	1,4
Average		60.6		**62.5**		



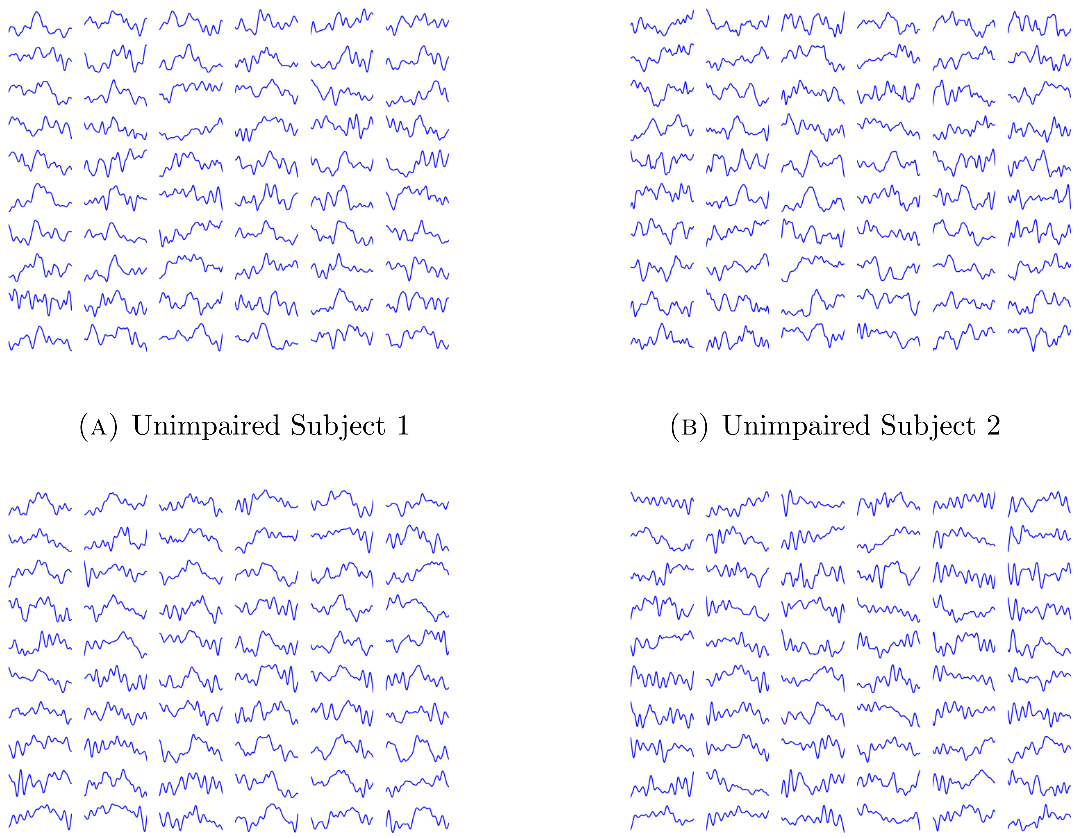
(A) Unimpaired Subject 1

(B) Unimpaired Subject 2

(C) Impaired Subject 1

(D) Impaired Subject 2

FIGURE 4.37. Biosemi Target Signals for all 4 Subjects before smoothing step



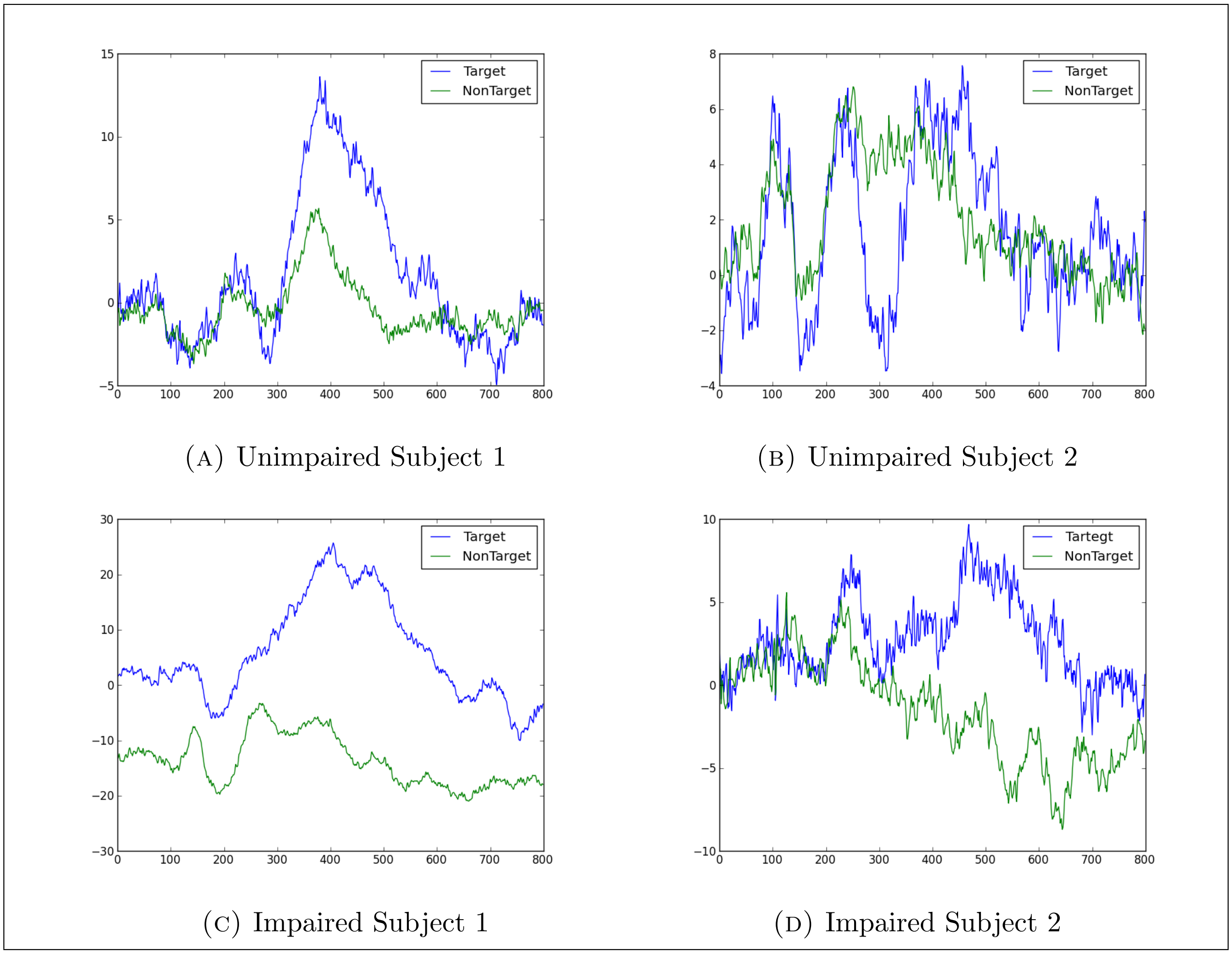
(A) Unimpaired Subject 1

(B) Unimpaired Subject 2

(C) Impaired Subject 1

(D) Impaired Subject 2

FIGURE 4.38. Biosemi Target Signals for all 4 Subjects after smoothing step



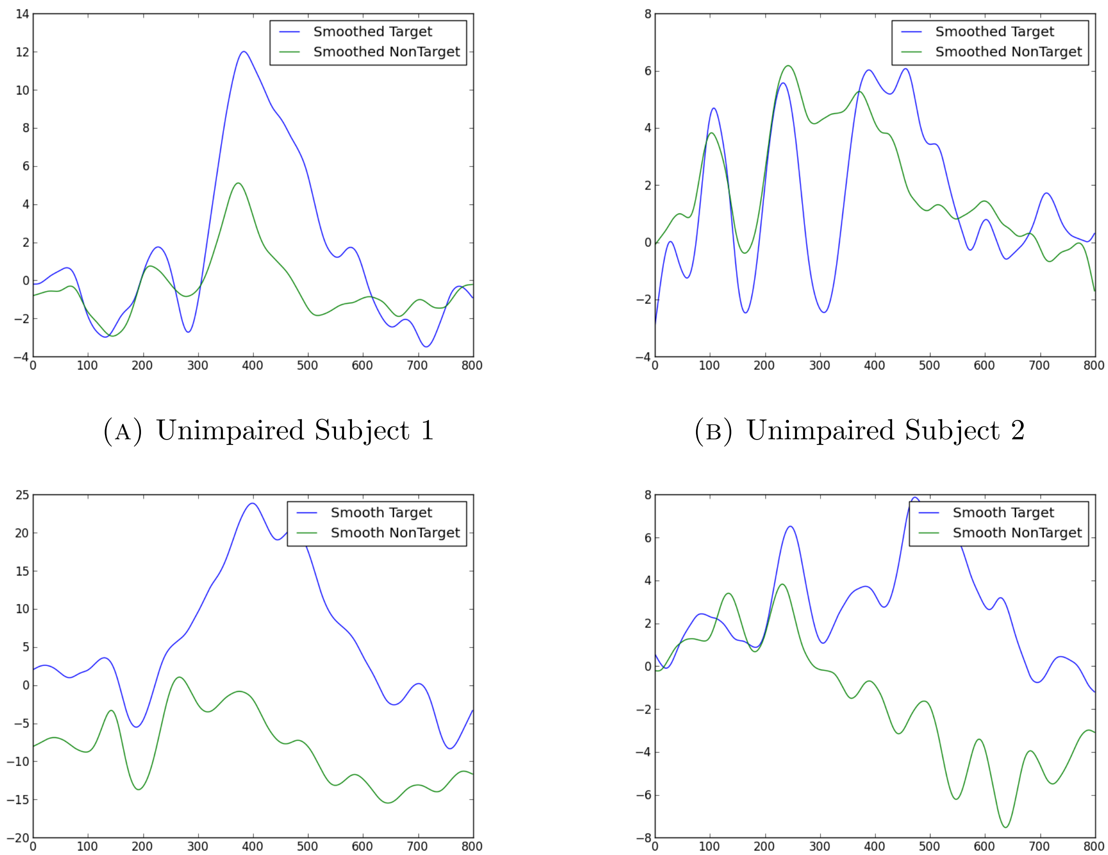
(A) Unimpaired Subject 1

(B) Unimpaired Subject 2

(C) Impaired Subject 1

(D) Impaired Subject 2

FIGURE 4.39. Biosemi Target and nonTarget means for all 4 Subjects before smoothing step



(A) Unimpaired Subject 1 (B) Unimpaired Subject 2

(C) Impaired Subject 1 (D) Impaired Subject 2

FIGURE 4.40. Biosemi Target and nonTarget means for all 4 Subjects after smoothing step

CHAPTER 5

Conclusion and Future Work

EEG was recorded from four subjects, two with motor impairment and two with no impairments. EEG data following presentations of single letters in the center of a computer display was collected while the subjects counted the occurrences of particular letters, in order to produce P300 waves. Three different experiments and scenarios were evaluated such as combine all of the three target letters and exclude some of the trials from this combination to be considered as a testing subspaces. In addition, creating the training subspaces using the data of two target letters and left one target letter as a testing subspaces to evaluate the classifier on more variations between the training and testing subspaces. Furthermore, examining the transfer information between subjects by creating the training subspaces using the data that are recorded from three different subjects and testing the method on the fourth subject.

Before the subspaces are created, a smoothing function is applied to filter the signals from the noise. Two different methods were tested, smoothing by regularization and bandpass filter. Typically bandpass filters are applied, but the smoothing algorithm demonstrated here does not rely on assumptions on the frequency content of EEG as required for bandpass filtering. No clear trends were found between these two methods. This study also examines ways of removing outliers that are based on principal angles. While the experiments show that the outlier removal methods did not help in this case, they may prove useful for other subjects or signals. This means that having an outlier as one of the trials that is used to create the subspace could not effect that much on the principal angles between this subspace and the other subspaces unlike other methods that depend on the trial averages concept.

In the first scenario, balanced classification accuracies of 90% and 74% were obtained for the unimpaired and 90% and 77% for the impaired subjects respectively, using one or two trials per subspace. In the second scenario, the accuracy was 83% and 85% for unimpaired subjects and 97% and 77% for impaired subjects using four trials per subspace. In the third scenario, having one or three training trials gives 82% and 69% as a balanced accuracy if the testing was on the unimpaired subjects and 87% and 66% accuracy if the testing was on the impaired subject. This accuracy is similar to other published studies on P300 detection. However, most other approaches use averages of more trials to obtain similar accuracies. The use of only a small number of trials makes this approach more appropriate for online application. Comparing the proposed method with the Linear Logistic Regression, principal angles method tends to give better results than the other method as an average of the all subjects. PA tends to produce results that are about 3% better than Linear Logistic Regression for the G.tec system and 2% using the Biosemi system.

Analyzing EEG patterns using principal angles is still a new direction in BCI field. Many studies can be implemented and added to this field to better analyze and classify the EEG patterns. Planned future work in this study includes the use of additional data sets recorded from other subjects, other recording systems with different sampling rates. As mentioned before in Chapter 2, the best accuracies in published results from the studies with the Competition data, some which are reviewed in this Chapter, are 40%-60% using single trial. The accuracies obtained in this dissertation are about 60%-80% for single trial. This warrants additional testing of the methods that were developed in this dissertation on the BCI Competition data [29]. Plans also include analyzing using additional channels and the use of other principal angle methods that are previously described in Chapter 2, such as the Discriminant Canonical Correlation (DCC) [23] method, that discriminates between subspaces by maximizing the canonical correlation within the subspace and minimizing it between the subspaces. In addition, applying EEG subspace analysis using the principal angles concept with online and mental tasks experiments and using the principal angles to find the best template in training subspaces that present the P300 clearly. Creating the subspaces by shifting in time, which means each trial contains information from the previous trial, might help to have more information in the trial and make the principal angle values correct to give better classification. Since most of the concentration in this dissertation is on the principal angles and analyzing the values of theses angles, studying and evaluating the principal vectors that are connected to these angles is a good direction to learn more deeply about how these vectors look like with the target P300 cases and non P300 trials. Furthermore, the similarities and differences of the vector variations within and across subspaces should be studied.

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